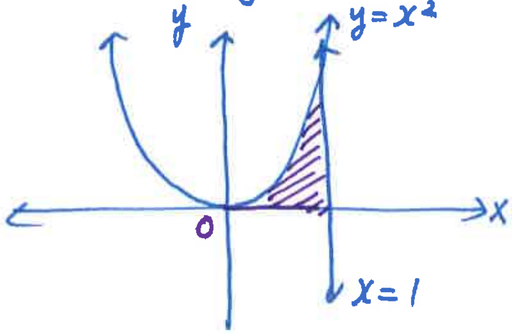
22A Area under a Curve $\Rightarrow$ FTC $\Rightarrow$ $\lim_{n\to\infty} \sum_{k=0}^{n} f(a+k\cdot\Delta x)\cdot\Delta x = \int_a^b f(x)\,dx$

$\Delta x = \frac{b-a}{n}$

1. Sketch the region bounded by $y = x^2$, the x-axis, and $x = 1$.

Find the exact area of this region.

$y=0$

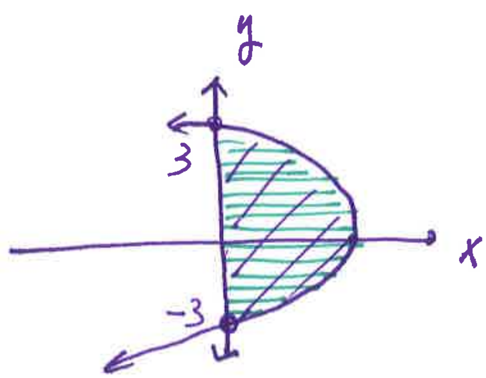


$$\Rightarrow \text{Area} = \int_0^1 x^2\,dx = \frac{1}{3}x^3\Big|_{x=0}^{x=1}$$

$$= \frac{1}{3}(1)^3 - \frac{1}{3}(0)^3 = \boxed{\frac{1}{3}}$$

2. Sketch the region bounded by $x = 9 - y^2$ and the y-axis.

Find the exact area of this region.



$0 = 9 - y^2$

$y = \pm 3$

$$\Rightarrow \text{Area} = \int_{-3}^{3} (9-y^2)\,dy = 2\int_0^3 (9-y^2)\,dy$$

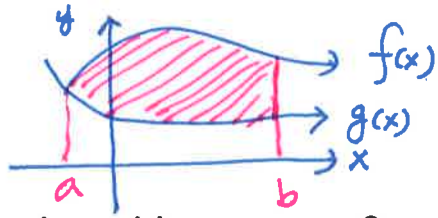
$$= 2\left[9y - \frac{1}{3}y^3\right]_{y=0}^{y=3}$$

$$= 2\left[9\cdot 3 - \frac{1}{\cancel{3}}\cancel{3^3}\right] = \cancel{8}$$

$\frac{27}{2}$

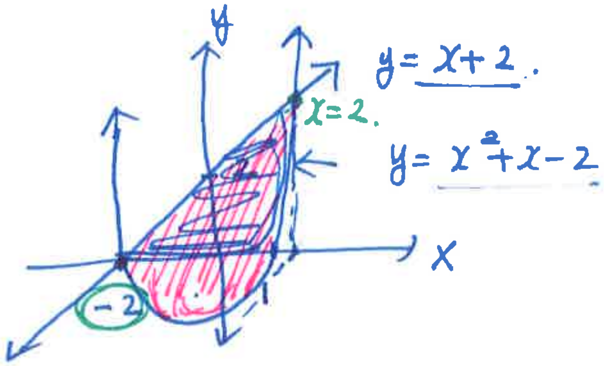
$$= \boxed{36}$$

$[27-9]\cdot 2.$

22B Area between Curves $\Rightarrow$ 

$\Rightarrow \int_a^b (f(x) - g(x))\,dx$.

1. Find the area of the region enclosed by $y = x + 2$ and $y = x^2 + x - 2$.



$y = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x = -2 \quad x = 1$

$\Rightarrow \cancel{x} + 2 = x^2 + \cancel{x} - 2$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = -2 \quad x = 2$

Area $\Rightarrow \int_{-2}^{2} [(x+2) - (x^2+x-2)]\,dx$.

$= \int_{-2}^{2} [-x^2+4]\,dx. = \left[-\frac{1}{3}x^3 + 4x\right]_{x=-2}^{x=2} = \left[-\frac{1}{3}(2)^3 + 4(2)\right] - \left[-\frac{1}{3}(-2)^3 + 4(-2)\right]$

48
-16

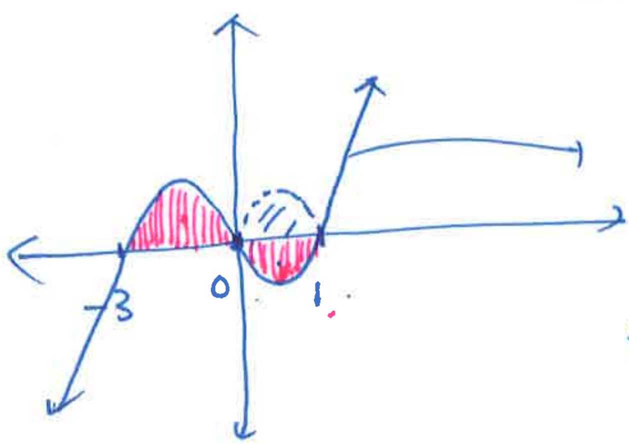
$16 - \frac{16}{3}$

$= 8 + 8 - \frac{8}{3} - \frac{8}{3} = \boxed{\frac{32}{3}}$

2. Find the total area of the regions contained by $f(x) = x^3 + 2x^2 - 3x$ and the x-axis.

$f(x) = x(x^2 + 2x - 3) = x(x+3)(x-1) = 0$

$x = 0, \quad x = 1, \quad x = -3$



Total Area $= \int_{-3}^{1} |x^3 + 2x^2 - 3x|\,dx$

$= \int_{-3}^{0} (x^3 + 2x^2 - 3x)\,dx + \int_{0}^{1} (-x^3 - 2x^2 + 3x)\,dx$

$= \left[\frac{1}{4}x^3 + \frac{2}{3}x^3 - \frac{3}{2}x^2\right]_{-3}^{0} + \left[-\frac{1}{4}x^3 - \frac{2}{3}x^3 + \frac{3}{2}x^2\right]_{x=0}^{x=1}$

$= 11.8\overline{3}$