Mathematical Exploration Paper
Stimulus: Games

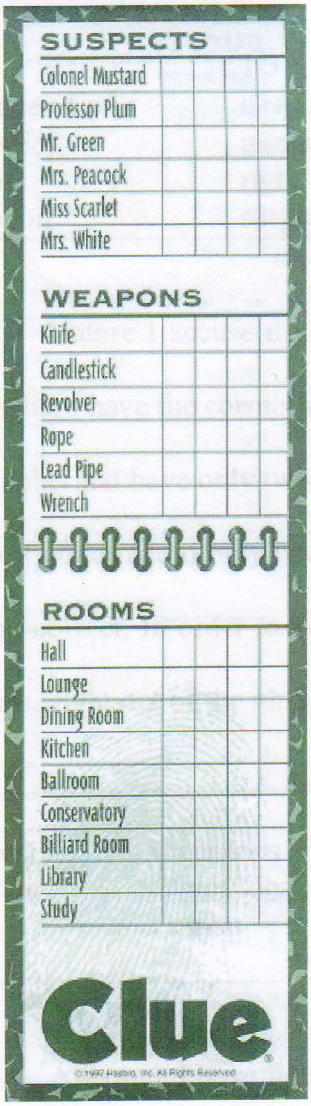
I knew my friend was a ruthless murderer from the beginning. Although I had a visceral feeling, I was not completely sure that it was she who committed the crime, what room the act took place in, nor the weapon used. It is my mission to investigate the case of the murder of Mr. Body and to prove my friend is a cold-blooded killer.

Parker Brothers' game "Clue" created in 1944, is a classic game of "who done it?"[1] The game is based off the murder of a "Mr. Body" and there are six suspects (Mrs. White, Mrs. Peacock, Miss Scarlet, Colonel Mustard, Professor Plum, and Mr. Green), six weapons (knife, revolver, lead pipe, candlestick, rope, and wrench), and nine rooms in which the murder may have taken place (ballroom, conservatory, library, billiard room, kitchen, dining room, study, lounge, and hall)[1]. There is one card for each suspect, weapon, and room to create a total of 21 cards. One of each type of card is randomly selected and placed in an envelope to create a full murder. The rest of the 18 cards are shuffled and divided between the players (in this case between my friend and me). Each player chooses to be a suspect and every turn each player guesses a possible scenario for the murder using the cards she got to eliminate what she knows must not be in the envelope. Each time I guess a scene my opponent must reveal if in her cards, she has at *least* one suspect, weapon or room I postulated[1]. The object of the game is to guess the correct scene before your opponent does.

Using the counting principle which states, "if the probability of one event occurring is *r* and the probability of another event occurring is *s*, then the probability of both events occurring is *rs*,"[3] I can figure out the probability of guessing the right scene without looking at my cards. There are six suspects, six weapons and nine rooms, therefore the probability of guessing correctly on my first try with no cards is 1/324.

1/6 Chance of guessing the right suspect	·	1/6 Chance of guessing the right weapon	·	1/9 Chance of guessing the right room	=	1/324 Chance of guessing the right scene

Although I do have superb guessing skills and am very suspicious that my opponent is the killer, I do not want to risk losing, so I will wait to see what cards I get first.



Out of the 18 cards left, my opponent and I each receive nine cards. In my hand, I received the rope, wrench, Mr. Green, Mrs. Peacock, Professor Plum, the hall, the library, the lounge, and the kitchen. I chose to be Mrs. Peacock, so I know that I am not the killer (like there was any doubt…) however my opponent, i.e. Miss Scarlet is not among my cards thus she is still a likely suspect. My probability of guessing the right scene now looks like this:

1/4 chance of guessing right weapon	·	1/3 chance of guessing right suspect	·	1/5 chance of guessing right room	=	1/60 chance of guessing correctly on my first turn

Before I accuse my opponent of being a vicious murderer, I want to be at least 50% sure that I have the correct scene. In order to do this I will have to be certain of two of the three variables and have only two possibilities left in the third variable to guess from. This is because once I am down to only two possibilities in the same category, I have a ½ chance, or 50%, of being correct. In order to be 50% sure, I must have 8 turns. I need to have five out of the six rooms, eight out of the nine rooms and four out of the six suspects to be 50% sure.

I need:	5 weapons		8 rooms		4 suspects	
I have:	-2 weapons		-4 rooms		-3 suspects	
	3 turns	+	4 turns	+	1 turn	= 8 turns

Although I have figured out the number of turns I will need to be 50% sure, I must also factor in independent events. Independent events are defined as: two events, A and B, whereas the event of A occurring does not affect the probability of B occurring. In the case of independent events the probabilities of both events occurring are multiplied, i.e. Probability(A and B) = $P(A) \cdot P(B)$.[5] In the process of taking my eight turns to be 50% sure, it is possible that I could guess correctly on my second turn, fifth turn, etc., it all depends on the three probabilities of guessing each characteristic of the murder correctly. Each of these probabilities are independent events, that is the chance of guessing the right room will not affect the chance of guessing the right weapon. Because each of these probabilities are independent, the number of

cards you are initially dealt for each category has a big impact on the overall probability of guessing right. For example, let's say instead I was dealt seven room cards, one weapon card, and one suspect card. My overall probability would look like this:

Formula for Independent Events:
$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

1/2 chance of guessing right room	·	1/5 chance of guessing right weapon	·	1/5 chance of guessing right suspect	=	1/50 chance of guessing correctly on first turn

Clearly, having a majority of cards in the biggest category doesn't give a very good shot at guessing correctly. On the other, having the majority in two categories, i.e. no rooms, four weapons and five suspects would turn out quite differently:

Formula for Independent Events:
$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

1/9 chance of guessing right room	·	1/2 chance of guessing right weapon	·	1 chance of guessing right suspect	=	1/18 chance of guessing correctly on first turn

If you recall in the hand I was dealt, there is, for the most part, an even spread of cards for each category which resulted in a one out of sixty chance in guessing correctly on my first turn. It seems that, although contrary to popular belief, having an even spread of cards gives the worst probability of guessing correctly, and having a majority of cards in the two smaller categories rather than the largest one gives the best chances.

As the accusations, eliminations and tension continued, it finally hit me. The murder of Mr. Body was performed in the ballroom with the revolver by none other than Miss Scarlet!

When my opponent had no cards to prove my indictment erroneous, I had the privilege to open the envelope and see that I was correct. Although the cards I was dealt did not give me very good chances to start out with, my instincts for justice (and perhaps just a little luck…) won me the game.

Works Cited

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