Exit slip #2 solutions

#1.

$$\left[\begin{array}{ccc|c} 2 & -1 & 5 & 5 \\ 1 & 1 & 1 & 4 \\ 1 & -4 & 6 & p \end{array}\right]$$

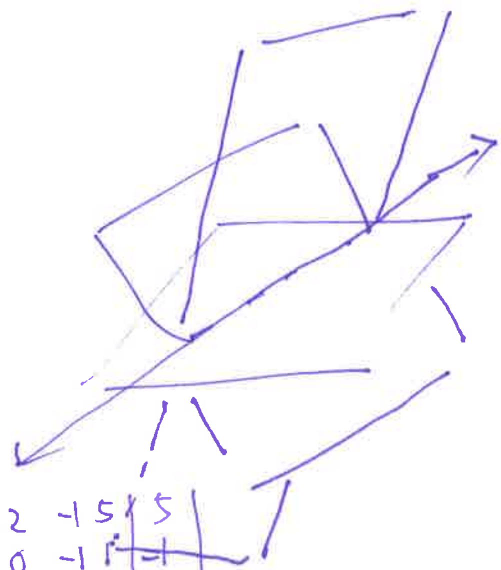
$R_2 - R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 2 & -1 & 5 & 5 \\ 1 & 1 & 1 & 4 \\ 0 & 5 & -5 & 4-p \end{array}\right]$$

$R_1 - 2R_2 \to R_2$

$5-8$

$$\left[\begin{array}{ccc|c} 2 & -1 & 5 & 5 \\ 0 & -3 & 3 & -3 \\ 0 & 5 & -5 & 4-p \end{array}\right]$$



$5R_2 + 3R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 2 & -1 & 5 & 5 \\ 0 & -3 & 3 & -3 \\ 0 & 0 & 0 & p+1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 2 & -1 & 5 & 5 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & p+1 \end{array}\right]$$

$-65 + 12 - 3p$

$-53 - 3p$

a) $p = -1 \Rightarrow p \neq -1 \Rightarrow \infty$ solution.

b) $z = t$ $\quad -y + t = -1 \quad \Rightarrow y = t + 1$

$2x - (t+1) + 5t = 5$

$2x - t - 1 + 5t = 5$

$\begin{array}{l} 15 \\ +13 \end{array}$

parametric

$$\begin{cases} x = -2t + 3 \\ y = t + 1 \\ z = t \end{cases}$$

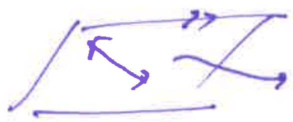
$2x = -4t + 6 \quad x = -2t + 3$

$-2t = x - 3$

$t = \dfrac{3-x}{2}$

Cartesian: $\boxed{z = y - 1 = \dfrac{3-x}{2}}$

c. $\longleftrightarrow z$ direction vector : $\begin{pmatrix}-2\\1\\1\end{pmatrix} d_1$



$\frac{x-4}{3} = y = \frac{z-5}{-1} \Rightarrow$ direction $\begin{pmatrix}3\\1\\-1\end{pmatrix} d_2$

$$d_1 \times d_2 = \begin{bmatrix} i & j & k \\ -2 & 1 & 1 \\ 3 & 1 & -1 \end{bmatrix} = i(-1-1) - j(2-3) + k(-2-3)$$
$$= -2i + j - 5k.$$

$$\vec{n}\cdot\vec{r} = \vec{a}\cdot\vec{n}$$

$$-2x+y-5k = \begin{pmatrix}4\\0\\5\end{pmatrix}\cdot\begin{pmatrix}-2\\1\\-5\end{pmatrix} = -8-25 = -33 \quad \therefore \boxed{-2x+y-5z=-33}$$

#2. a)
① $cis\left(\frac{2\pi}{9}\right)$

② $cis\left(\frac{4\pi}{9}\right)$

③ $cis\left(\frac{6\pi}{9}\right) = cis\left(\frac{2\pi}{3}\right)$

④ $cis\left(\frac{8\pi}{9}\right)$

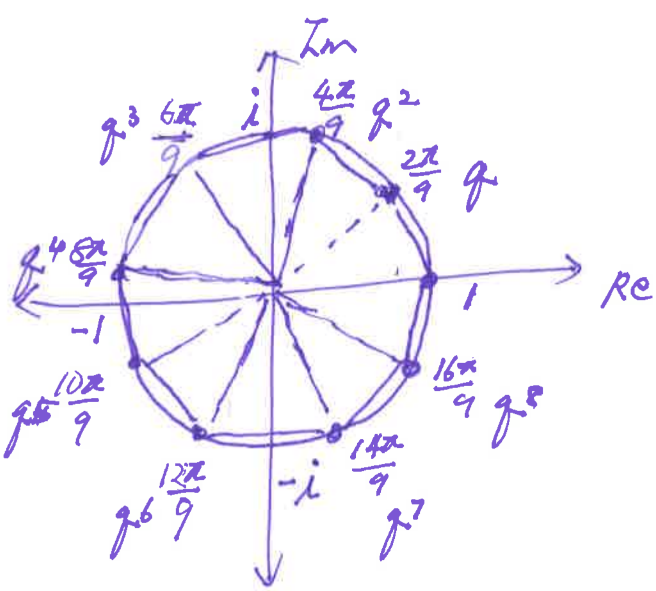
⑤ $cis\left(\frac{10\pi}{9}\right)$

⑥ $cis\left(\frac{12\pi}{9}\right) = cis\left(\frac{4\pi}{3}\right)$

⑦ $cis\left(\frac{14\pi}{9}\right)$

⑧ $cis\left(\frac{16\pi}{9}\right)$

⑨ $cis\left(\frac{18\pi}{9}\right) = cis(2\pi)$



b) $q = cis\left(\frac{2\pi}{9}\right) = \cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)$

$q^2 = \left[cis\left(\frac{4\pi}{9}\right)\right] = \cos\left(\frac{4\pi}{9}\right) + i\sin\left(\frac{4\pi}{9}\right)$

$\vdots$

$q^8 = \left[cis\,\frac{16\pi}{9}\right] = \cos\left(\frac{16\pi}{9}\right) + i\sin\left(\frac{16\pi}{9}\right)$

(3)

q & q^8 are conjugates. $\Rightarrow$ $q = \cos\left(\frac{2\pi}{9}\right) + i\sin\left(\frac{2\pi}{9}\right)$

q^2 & q^7 are conjugates $\quad q^8 = \cos\left(\frac{2\pi}{9}\right) - i\sin\left(\frac{2\pi}{9}\right)$

q^3 & q^6 " $\quad q^2 = \cos\left(\frac{4\pi}{9}\right) + i\sin\left(\frac{4\pi}{9}\right)$

q^4 and q^5 " $\quad q^7 = \cos\left(\frac{4\pi}{9}\right) - i\sin\left(\frac{4\pi}{9}\right)$

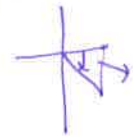
$q^3 = -\left[\cos\left(\frac{\pi}{3}\right) - i\sin\left(\frac{\pi}{3}\right)\right]$

$q^6 = -\left[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right]$

$\therefore\ (q + q^2 + q^3 + q^4)$ $\quad q^4 = -\left[\cos\left(\frac{\pi}{9}\right) - i\sin\left(\frac{\pi}{9}\right)\right]$

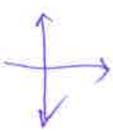
is a conjugate of $\quad q^5 = -\left[\cos\left(\frac{\pi}{9}\right) + i\sin\left(\frac{\pi}{9}\right)\right]$

$(q^5 + q^6 + q^7 + q^8)$

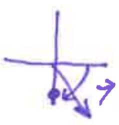


$-\frac{\pi}{4}$ OR $\frac{7\pi}{4}$

\# 3. $\left[16\sqrt{2} - 16i\sqrt{2}\right]^{\frac{1}{5}} = \left[32 \operatorname{cis}\left(\frac{7\pi}{4} + 2\pi k\right)\right]^{\frac{1}{5}}$



$r = \sqrt{16^2 \cdot 2 + 16^2 \cdot 2} = 16\sqrt{2+2} = 32.$



$\theta = -\frac{\pi}{4}$ $\qquad \sqrt[5]{32} = \sqrt[5]{2^5} = 2$

① $2 \cdot \operatorname{cis}\left(\frac{7\pi}{20}\right)$ OR $2\operatorname{cis}\left(\frac{-\pi}{20}\right)$

② $2\operatorname{cis}\left(-\frac{\pi}{20} + \frac{2\pi}{5 \cdot 4}\right) = 2\operatorname{cis}\left(\frac{7\pi}{20}\right)$

$\cdot 16$

③ $2\operatorname{cis}\left(\frac{-\pi}{20} + \frac{4\pi}{5}\right) = 2\operatorname{cis}\left(\frac{15\pi}{20}\right)$ $\frac{8\pi}{20}$

④ $2\operatorname{cis}\left(\frac{-\pi}{20} + \frac{6\pi}{5 \cdot 4}\right) = 2\operatorname{cis}\left(\frac{23\pi}{20}\right)$

⑤ $2\operatorname{cis}\left(\frac{-\pi}{20} + \frac{8\pi}{5 \cdot 4}\right) = 2\operatorname{cis}\left(\frac{31\pi}{20}\right)$

\# 4. $-b = (3+i)+(3-i) = 6$

$\boxed{b = -6}$

$c = (3-i)(3+i)$

$\boxed{c = 9 + 1 = 10}$