IB Math HL 1 The Chain Rule

$\frac{dy}{dx}$

Warm up: Suppose $f(x)=x^2+(b+1)x+2c$, $f(2)=4$ and $f'(-1)=2$. Find the constants b and c.

$f(2)=2^2+(b+1)2+2c=4$

$\cancel{4}+2b+2+2c=\cancel{4}$

$2b+2c=-2$

$b+c=-1$

$f'(x)=2x+b+1$

$f'(-1)=2(-1)+b+1=2$ $\boxed{b=3}$

$\boxed{c=-4}$

1. Find $g(x)$ and $f(x)$ such that $f(g(x))=\dfrac{10}{(3x-x^2)^3}$

$\boxed{f(x)=\frac{10}{x^3} \quad g(x)=3x-x^2}$

2. a. Differentiate with respect to x: $y=(5x^2+3)^2$

Find $\frac{dy}{dx}$

$y=25x^4+30x^2+9$

$y'=\boxed{\frac{dy}{dx}=100x^3+60x}$

b. Is $\dfrac{dy}{dx}=2(5x^2+3)$?

$=10x^2+6$

NO.

c. Find a way to write $\dfrac{dy}{dx}$ using $2(5x^2+3)$.

$\frac{dy}{dx}=2(5x^2+3)'[10x]$

$y=(5x^2+3)^2$

3. a. If $y=(x^2+3x)^2$ and $u=x^2+3x$, write y in terms of u.

$y=(u)^2$

$u(x)=x^2+3x$

b. Find $\dfrac{dy}{du}$ and $\dfrac{du}{dx}$.

$\frac{dy}{du}=2u$ $\quad \frac{du}{dx}=2x+3$

c. Write $\dfrac{dy}{dx}$ in terms of $\dfrac{dy}{du}$ and $\dfrac{du}{dx}$.

$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$

$=2u\cdot(2x+3)$

$=2(x^2+3x)(2x+3)$

The Chain Rule

If $y = f(u)$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

If $y = f(g(x))$, then $\frac{df}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$.

If $y = (f(x))^n$, then $\frac{dy}{dx} = n\,(f(x))^{n-1} \cdot \frac{df}{dx}$.

Find $\frac{dy}{dx}$:

4. $y = (x^2 - 2x)^4$

$$\boxed{\frac{dy}{dx} = 4(x^2 - 2x)^3 (2x - 2)}$$

5. $y = \frac{4}{\sqrt{1 - 2x^3}} = 4(1 - 2x^3)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = \cancel{4}\left(-\frac{1}{\cancel{2}}\right)(1 - 2x^3)^{-\frac{3}{2}}(-6x^2)$$

$$= \boxed{\frac{12x^2}{(1 - 2x^3)^{3/2}}} \checkmark$$

$$= \boxed{\frac{12x^2}{(1 - 2x^3)\sqrt{1 - 2x^3}}} \checkmark$$

6. $y = \sqrt[3]{x^4 - x} = (x^4 - x)^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3}(x^4 - x)^{-\frac{2}{3}}(4x^3 - 1)$$

$$\boxed{\frac{4x^3 - 1}{3(x^4 - x)^{2/3}}}$$

$$= \boxed{\frac{4x^3 - 1}{3\sqrt[3]{(x^4 - x)^2}}}$$

Find the gradient at x=1. $y = (x^2 - 2x)^4$