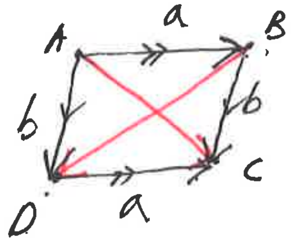
Exit Slip #3.

1. Parallelogram ABCD is formed by vectors $\vec{AB} = \vec{DC} = a$ and $\vec{AD} = \vec{BC} = b$

a) Write the diagonals $\vec{AC}$ and $\vec{BD}$ in terms if a and b.



$\vec{AC} = a + b$

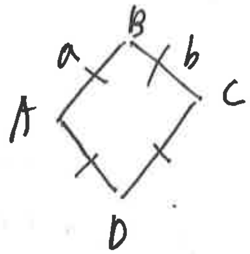
$\vec{BD} = b - a$.

b) Calculate the ~~vector~~ Dot product $\vec{AC}$ and $\vec{BD}$ in terms of a and b.

$\vec{AC} \cdot \vec{BD} = (a+b)\cdot(b-a) = a\cdot b - a\cdot a + b\cdot b - b\cdot a$

$= \cancel{a\cdot b} - a\cdot a + b\cdot b - \cancel{a\cdot b} = -a\cdot a + b\cdot b.$

c) Show that if ABCD is a rhombus, then $\vec{AC}$ is perpendicular to $\vec{BD}$.



$\Rightarrow |a| = |b|$

$\Rightarrow \vec{AC}\cdot\vec{BD} = -a\cdot a + b\cdot b = 0 \Rightarrow \boxed{\therefore \vec{AC} \perp \vec{BD}}$

4. Given three points: A (1, 4, 2), B (0, 8, 0), and C(4, 10, 3).

a. Find $\overrightarrow{AB}\bullet\overrightarrow{AC}$

$\vec{AB} = \begin{pmatrix} -1 \\ 4 \\ -2 \end{pmatrix}$ $\vec{AC} = \begin{pmatrix} 3 \\ 6 \\ 1 \end{pmatrix}$

b. Find the angle $B\hat{A}C$.

$\theta = B\hat{A}C = \arccos\left[\frac{\vec{AB}\cdot\vec{AC}}{|AB|\,|AC|}\right] = \arccos\left[\frac{19}{\sqrt{21}\sqrt{46}}\right] \approx 52.3^\circ$

c. Find the area of the triangle ABC by two different methods.

① Area of $\Delta = \frac{\sqrt{21}\sqrt{46}\sin 52.3^\circ}{2}$

≈ 12.3

② $\vec{AB}\times\vec{AC} \Rightarrow \begin{vmatrix} i & j & k \\ -1 & 4 & -2 \\ 3 & 6 & 1 \end{vmatrix}$ $-j(-1+6)$

$= 16i + 5j - 18k$

Area of $\Delta = \frac{1}{2}\sqrt{16^2 + 5^2 + 18^2}$

≈ 12.3