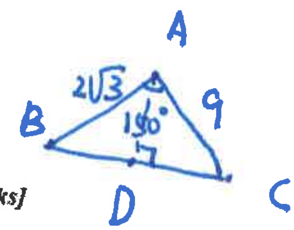
Ch 14. IB Questions (key)



#1

(a) $BC^2 = 12 + 81 + 2 \times 2\sqrt{3} \times 9 \times \frac{\sqrt{3}}{2} = 147$ *M1A1*

$BC = 7\sqrt{3}$ *A1*

[3 marks]

(b) area of triangle $ABC = \frac{1}{2} \times 9 \times 2\sqrt{3} \times \frac{1}{2}\ \left(= \frac{9\sqrt{3}}{2}\right)$ *M1A1*

therefore $\frac{1}{2} \times AD \times 7\sqrt{3} = \frac{9\sqrt{3}}{2}$ *M1*

$AD = \frac{9}{7}$ *A1*

[4 marks]

Total [7 marks]

#2

taking cross products with $\boldsymbol{a}$, *M1*

$\boldsymbol{a} \times (\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \times \boldsymbol{0} = \boldsymbol{0}$ *A1*

using the algebraic properties of vectors and the fact that $\boldsymbol{a} \times \boldsymbol{a} = \boldsymbol{0}$, *M1*

$\boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{a} \times \boldsymbol{c} = \boldsymbol{0}$ *A1*

$\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{c} \times \boldsymbol{a}$ *AG*

taking cross products with $\boldsymbol{b}$, *M1*

$\boldsymbol{b} \times (\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{0}$

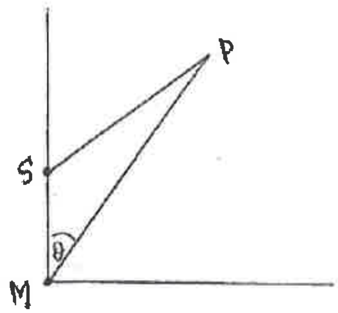
$\boldsymbol{b} \times \boldsymbol{a} + \boldsymbol{b} \times \boldsymbol{c} = \boldsymbol{0}$ *A1*

$\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{b} \times \boldsymbol{c}$ *AG*

this completes the proof

[6 marks]

#3



θ : Northeast

(a) let the interception occur at the point P, t hrs after 12:00
then, $\text{SP} = 20t$ and $\text{MP} = 30t$ *A1*
using the sine rule,

$$\frac{\text{SP}}{\text{MP}} = \frac{2}{3} = \frac{\sin\theta}{\sin 135}$$ *M1A1*

whence $\theta = 28.1$ *A1*

[4 marks]

(b) using the sine rule again,

$$\frac{\text{MP}}{\text{MS}} = \frac{\sin 135}{\sin(45 - 28.1255...)}$$ *M1A1*

$$30t = 10 \times \frac{\sin 135}{\sin 16.8745...}$$ *M1*

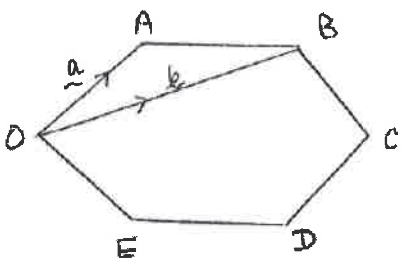
$$t = 0.81199...$$ *A1*

the interception occurs at 12:49 *A1*

[5 marks]

Total [9 marks]

#4



(a) $\mathrm{OC} = \mathrm{AB} + \mathrm{OA}\cos 60 + \mathrm{BC}\cos 60$ *M1*

$= \mathrm{AB} + \mathrm{AB} \times \frac{1}{2} + \mathrm{AB} \times \frac{1}{2}$ *A1*

$= 2\mathrm{AB}$ *AG*

[2 marks]

(b) $\vec{\mathrm{OC}} = 2\vec{\mathrm{AB}} = 2(b - a)$ *M1A1*

$\vec{\mathrm{OD}} = \vec{\mathrm{OC}} + \vec{\mathrm{CD}}$ *M1*

$= \vec{\mathrm{OC}} + \vec{\mathrm{AO}}$ *A1*

$= 2b - 2a - a = 2b - 3a$ *A1*

$\vec{\mathrm{OE}} = \vec{\mathrm{BC}}$ *M1*

$= 2b - 2a - b = b - 2a$ *A1*

[7 marks]

Total [9 marks]

#5

(a) (i) $\vec{\mathrm{OA}} \times \vec{\mathrm{OB}} = i + 7j - 5k$ *A1*

(ii) area $= \frac{1}{2}\left| i + 7j - 5k \right| = \frac{5\sqrt{3}}{2}$ (4.33) *M1A1*

4

#6 (a) any attempt to use sine rule **M1**

$$\frac{\text{AB}}{\sin\frac{\pi}{3}} = \frac{\sqrt{3}}{\sin\left(\frac{2\pi}{3}-\theta\right)}$$ **A1**

$$= \frac{\sqrt{3}}{\sin\frac{2\pi}{3}\cos\theta - \cos\frac{2\pi}{3}\sin\theta}$$ **A1**

Note: Condone use of degrees.

$$= \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta}$$ **A1**

$$\frac{\text{AB}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta}$$

$$\therefore \text{AB} = \frac{3}{\sqrt{3}\cos\theta + \sin\theta}$$ **AG**

[4 mark

(b) **METHOD 1**

$$(\text{AB})' = \frac{-3\left(-\sqrt{3}\sin\theta + \cos\theta\right)}{\left(\sqrt{3}\cos\theta + \sin\theta\right)^2}$$ **M1A1**

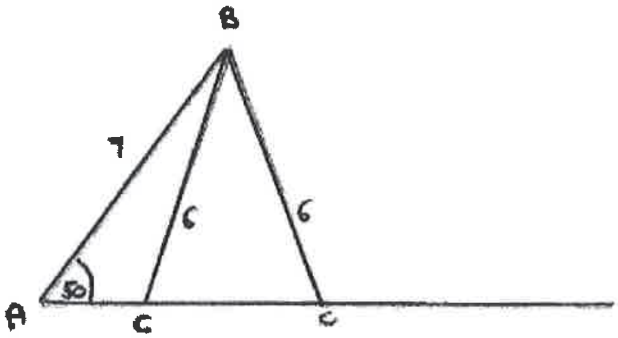
setting $(\text{AB})' = 0$ **M1**

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \frac{\pi}{6}$$ **A1**

(5)

#7



METHOD 1

$$\frac{6}{\sin 50} = \frac{7}{\sin C} \Rightarrow \sin C = \frac{7 \sin 50}{6}$$ *(M1)*

$C = 63.344...$ *(A1)*

or $C = 116.655...$ *(A1)*

$B = 13.344...$ (or $B = 66.656...$) *(A1)*

area $= \frac{1}{2} \times 6 \times 7 \times \sin 13.344...$ (or $\frac{1}{2} \times 6 \times 7 \times \sin 66.656...$) *(M1)*

$4.846...$ (or $= 19.281...$)

so answer is 4.85 (cm^2) ***A1***

#8 (a) require $\begin{pmatrix} 4 \\ \lambda \\ 10 \end{pmatrix} = s \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$ *(M1)*

$\Rightarrow 4 = 2s \Rightarrow s = 2 \Rightarrow \lambda = 6$ ***A1***

Note: Accept cross product solution.

[2 marks]

(b) require $v \bullet w = 2 \times 4 + 3 \times \lambda + 5 \times 10 = 0 \Rightarrow 3\lambda = -58 \Rightarrow \lambda = \frac{-58}{3} (-19.3)$ ***M1A1***

[2 marks]

(c) $v \bullet w = 2 \times 4 + 3 \times \lambda + 5 \times 10 = \sqrt{2^2 + 3^2 + 5^2} \times \sqrt{4^2 + \lambda^2 + 10^2} \times \cos 10°$ *(M1)(A1)*

$58 + 3\lambda = \sqrt{38} \times \sqrt{116 + \lambda^2} \times \cos 10°$

$\lambda = 3.73$ or 8.76 ***A1A1***

[4 marks]

Total [8 marks]

#9

(a) $u \times v = \begin{pmatrix} 2(0)+2b \\ -2a-1(0) \\ b-2a \end{pmatrix} = \begin{pmatrix} 2b \\ -2a \\ b-2a \end{pmatrix}$ **(M1)(A1)**

$\begin{pmatrix} 2b \\ -2a \\ b-2a \end{pmatrix} = \begin{pmatrix} 4 \\ b \\ c \end{pmatrix}$ **(M1)**

$\Rightarrow a=-1,\ b=2,\ c=4$ **A2**

Note: Award ***A1*** for two correct.

[5 marks]

(4)

#10

(a) $\overrightarrow{PR} = \boldsymbol{a}+\boldsymbol{b}$ ***A1***

$\overrightarrow{QS} = \boldsymbol{b}-\boldsymbol{a}$ ***A1***

[2 marks]

(b) $\overrightarrow{PR}\cdot\overrightarrow{QS} = (\boldsymbol{a}+\boldsymbol{b})\cdot(\boldsymbol{b}-\boldsymbol{a})$ ***M1***

$= |\boldsymbol{b}|^2 - |\boldsymbol{a}|^2$ ***A1***

for a rhombus $|\boldsymbol{a}| = |\boldsymbol{b}|$ ***R1***

hence $|\boldsymbol{b}|^2 - |\boldsymbol{a}|^2 = 0$ ***A1***

Note: Do not award the final ***A1*** unless ***R1*** is awarded.

hence the diagonals intersect at right angles ***AG***

[4 marks]

Total [6 marks]