Chapter 18 Review Solutions

1. $f'(x) = \frac{20}{3}x^{-\frac{1}{6}} - 63x^{8} - 13\pi x^{-\pi-1}$

2. $f'(x) = \cos(3x^2 + 5x - 7)(6x + 5)$

3. $f'(x) = 35x^6 \ln(4x^5 - 13x) + (5x^7)\frac{20x^4 - 13}{4x^5 - 13x}$

4. $f'(x) = 12(3x+1)^3 \sec^2((3x+1)^4)\, e^{\tan((3x+1)^4)}$

5. $f'(x) = (\cos^2 x^2 - \sin^2 x^2)\, 2x$

6. $f'(x) = -9\cos^8(2x^4 - 3x)\sin(2x^4 - 3x)(8x^3 - 3)$

7. $f'(x) = \frac{12x^3 + 5}{3x^4 + 5x}$

8. $f'(x) = 5\left(\frac{2 + e^{7x}}{x^2 - 8x}\right)^4 \left(\frac{(x^2 - 8x)(7e^{7x}) - (2 + e^{7x})(2x - 8)}{(x^2 - 8x)^2}\right)$

9. $f'(x) = -\frac{2}{3}x(8 - x^2)^{-\frac{2}{3}}\sin x^3 + 3x^2\sqrt[3]{8 - x^2}\cos x^3$

10. $f'(x) = \frac{(\sec 7x)(9x^8 - 8) - 7(x^9 - 8x)\sec 7x \tan 7x}{(\sec 7x)^2}$

11. $f'(x) = \frac{3x^2}{(x^3 - 8)\ln 7}$

12. $f'(x) = 9^{x^8 + 2x} \ln 9\,(8x^7 + 2)$

13. $f(x) = \sin^{-1}\left(\frac{x}{3}\right)$

$$f'(x) = \frac{\frac{1}{3}}{\sqrt{1-\left(\frac{x}{3}\right)^2}} = \boxed{\frac{1}{3\sqrt{1-\frac{x^2}{9}}}}$$

14. $f'(x) = 5^x \ln 5 \cos^{-1} x + 5^x \cdot \frac{-1}{\sqrt{1-x^2}}$

15. $y = (\tan^{-1}(5x))^3$

$$\boxed{\frac{dy}{dx} = 3(\tan^{-1}(5x))^2 \cdot \frac{5}{1+25x^2}}$$

16. $f'(x) = \frac{1}{2}(\sec^{-1} x)^{-\frac{1}{2}} \frac{1}{|x|\sqrt{x^2-1}}$

17. $6xy + 3x^2 \frac{dy}{dx} - y - x\frac{dy}{dx} + 5e^{y^2} 2y \frac{dy}{dx} \cot x + 5e^{y^2}(-\csc^2 x) = 0$

$$\frac{dy}{dx} = \frac{y - 6xy + 5e^{y^2}\csc^2 x}{3x^2 - x + 10y e^{y^2} \cot x}$$

18. $1 \cdot \sin y + x \cos y \frac{dy}{dx} - \sin 2y \cdot 2\frac{dy}{dx} = 4x$

$$\frac{dy}{dx} = \frac{4x - \sin y}{x\cos y - 2\sin 2y}$$

19. $f'(x) = \sec^2 4x \cdot 4$

$$f'\left(\frac{5\pi}{24}\right) = 4\sec^2\left(4 \cdot \frac{5\pi}{24}\right)$$

$$= 4\sec^2\left(\frac{5\pi}{6}\right)$$

$$= 4\left(-\frac{2}{\sqrt{3}}\right)^2$$

$$= 4 \cdot \frac{4}{3} = \boxed{\frac{16}{3}}$$

20. $3x^2y - 2xy^2 = 1$

$x = 1 \rightarrow 3y - 2y^2 = 1$

$0 = 2y^2 - 3y + 1$

$0 = (2y - 1)(y - 1)$

$y = \frac{1}{2}, 1 \rightarrow (1, \frac{1}{2}), (1, 1)$

$6xy + 3x^2\frac{dy}{dx} - 2y^2 - 2x \cdot 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{2y^2 - 6xy}{3x^2 - 4xy}$

$\left.\frac{dy}{dx}\right|_{(1, \frac{1}{2})} = \frac{2(\frac{1}{2})^2 - 6(1)(\frac{1}{2})}{3(1)^2 - 4(1)(\frac{1}{2})} = \boxed{-\frac{5}{2}}$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{2(1)^2 - 6(1)(1)}{3(1)^2 - 4(1)(1)} = \boxed{4}$

21. $f(x) = ax^3 + bx + 2$

$f(2) = 28$

$28 = a(2)^3 + b(2) + 2$

$26 = 8a + 2b$

$26 = 8a + 2(45 - 12a)$

$a = 4$

$f'(x) = 3ax^2 + b$

$f'(2) = 45$

$45 = 3a(2)^2 + b$

$45 = 12a + b$

$45 - 12a = b$

$45 - 12(4) = b$

$-3 = b$

$\boxed{a = 4,\ b = -3}$

22. $x^2 - xy + y^2 = 4$

$2x - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$(2y-x)\frac{dy}{dx} = y - 2x$

$\frac{dy}{dx} = \frac{y-2x}{2y-x}$

$\frac{d^2y}{dx^2} = \frac{(2y-x)\left(\frac{dy}{dx} - 2\right) - (y-2x)\left(2\frac{dy}{dx} - 1\right)}{(2y-x)^2}$

$$\frac{d^2y}{dx^2} = \frac{(2y-x)\left(\frac{y-2x}{2y-x} - 2\right) - (y-2x)\left(\frac{2(y-2x)}{2y-x} - 1\right)}{(2y-x)^2}$$

23. $x^2 + 4y^2 = 17$

$2x + 8y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{4y}$

$\frac{d^2y}{dx^2} = \frac{4y(-1) - (-x)\,4\frac{dy}{dx}}{(4y)^2}$

$$\frac{d^2y}{dx^2} = \frac{-4y + 4x\left(\frac{-x}{4y}\right)}{(4y)^2}$$