Chapter 19 Review

1. $3x^2y - 2xy^2 = 1$

$\frac{d}{dx}(3x^2y - 2xy^2) = \frac{d}{dx}(1)$

$6xy + 3x^2\frac{dy}{dx} - 2y^2 - 2x\cdot 2y\frac{dy}{dx} = 0$

$(3x^2 - 4xy)\frac{dy}{dx} = 2y^2 - 6xy$

$\frac{dy}{dx} = \frac{2y^2 - 6xy}{3x^2 - 4xy}$

$x = 1 \rightarrow 3y - 2y^2 = 1$

$0 = 2y^2 - 3y + 1$

$0 = (2y-1)(y-1)$

$y = \frac{1}{2}, 1$

$\left.\frac{dy}{dx}\right|_{(1,\frac{1}{2})} = \frac{2(\frac{1}{2})^2 - 6(1)(\frac{1}{2})}{3(1)^2 - 4(1)(\frac{1}{2})} = \frac{\frac{1}{2} - 3}{3-2} = -\frac{5}{2}$

$\boxed{y = -\frac{5}{2}(x-1) + \frac{1}{2}}$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{2(1)^2 - 6(1)(1)}{3(1)^2 - 4(1)(1)} = \frac{2-6}{3-4} = +4$

$\boxed{y = +4(x-1) + 1}$

2. $xy^2 = x^2y - 2$

$\frac{d}{dx}(xy^2) = \frac{d}{dx}(x^2y - 2)$

$y^2 + x\cdot 2y\frac{dy}{dx} = 2xy + x^2\frac{dy}{dx}$

$(2xy - x^2)\frac{dy}{dx} = 2xy - y^2$

$\frac{dy}{dx} = \frac{2xy - y^2}{2xy - x^2}$

$x = 2 \rightarrow 2y^2 = 4y - 2$

$2y^2 - 4y + 2 = 0$

$2(y^2 - 2y + 1) = 0$

$2(y-1)^2 = 0$

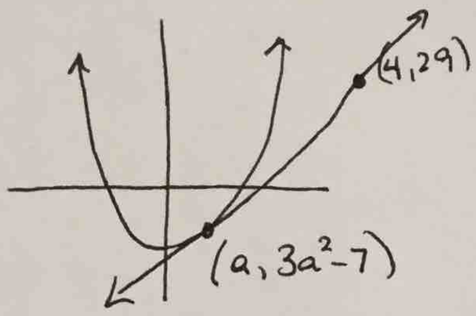
$y = 1$

slope of tangent

$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{2\cdot 2\cdot 1 - 1^2}{2\cdot 2\cdot 1 - 2^2} =$ undefined $\therefore$ slope of normal $= 0$

normal line : $\boxed{y = 1}$

3. $f(x) = 3x^2 - 7$



slope of this tangent $= \dfrac{29 - (3a^2 - 7)}{4 - a}$

slope of this tangent $= f'(a) = 6a$

$$6a = \frac{29 - (3a^2 - 7)}{4 - a}$$

$$6a = \frac{36 - 3a^2}{4 - a}$$

$$24a - 6a^2 = 36 - 3a^2$$

$$0 = 3a^2 - 24a + 36$$

$$0 = 3(a^2 - 8a + 12)$$

$$0 = 3(a - 6)(a - 2)$$

$$a = 6, 2$$

$a = 6 \rightarrow f'(6) = 36$

$$\boxed{y = 36(x - 4) + 29}$$

$a = 2 \rightarrow f'(2) = 12$

$$\boxed{y = 12(x - 4) + 29}$$

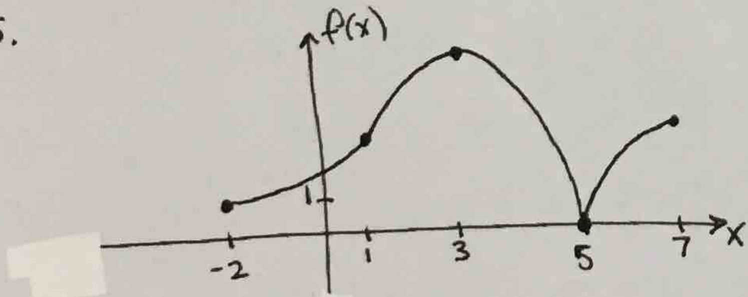
4. a. $x = 0, 5$

* Think about the behavior of f near the endpoints.

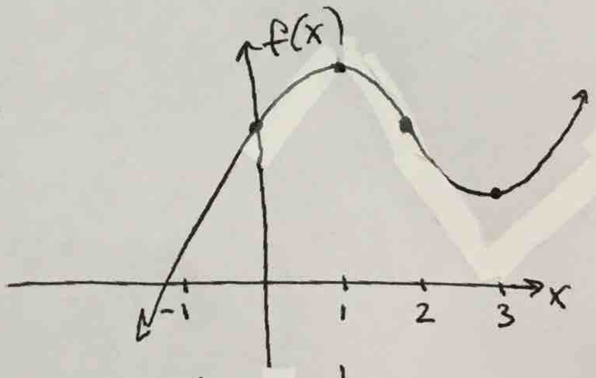
b. $x = -1, \frac{7}{3}$

c. $x = 1, 3$

5.



6.



* Note where my graphs are concave up/down!
Does your graph clearly show concavity/points of inflection, max, min, stationary points, increasing, and decreasing?

7a. $f(x) = 6\sqrt[3]{x}$, $[0, 27]$

$f'(x) = 2x^{-2/3}$, $f(27) = 6\sqrt[3]{27} = 18$

$f(0) = 0$

$\frac{2}{\sqrt[3]{c^2}} = \frac{18-0}{27-0}$

$\frac{2}{\sqrt[3]{c^2}} = \frac{2}{3}$

$\sqrt[3]{c^2} = 3$

$c^2 = 27$

$c = \sqrt{27}$

$\boxed{c = 3\sqrt{3}}$

b. $f(x) = \sqrt{2-x}$, $[-7, 2]$

$f'(x) = \frac{1}{2}(2-x)^{-\frac{1}{2}}(-1)$ $f(2) = 0$

$f(-7) = 3$

$f'(x) = \frac{-1}{2\sqrt{2-x}}$

$\frac{-1}{2\sqrt{2-c}} = \frac{3-0}{-7-2}$

$\frac{-1}{2\sqrt{2-c}} = -\frac{1}{3}$

$2\sqrt{2-c} = 3$

$\sqrt{2-c} = \frac{3}{2}$

$2-c = \frac{9}{4}$

$2 - \frac{9}{4} = c$

$\boxed{-\frac{1}{4} = c}$

c. $f(x) = e^{2x} + 4$, $[0, \ln 3]$

$f'(x) = 2e^{2x}$ $f(\ln 3) = e^{2\ln 3} + 4 = e^{\ln 9} + 4 = 9 + 4 = 13$

$f(0) = e^0 + 4 = 5$

$2e^{2c} = \frac{13-5}{\ln 3 - 0}$

$2e^{2c} = \frac{8}{\ln 3}$

$e^{2c} = \frac{4}{\ln 3}$

$2c = \ln\left(\frac{4}{\ln 3}\right)$

$\boxed{c = \frac{1}{2}\ln\left(\frac{4}{\ln 3}\right)}$

8. a. $f(x) = \begin{cases} 2x^3 & , x \le 4 \\ a(x-3)^2 + b & , x > 4 \end{cases}$

$f'(x) = \begin{cases} 6x^2 & , x < 4 \\ 2a(x-3) & , x > 4 \end{cases}$

$2(4)^3 = a(4-3)^2 + b$

$128 = a + b$

$128 = 48 + b$

$80 = b$

$\boxed{\begin{array}{l} a = 48 \\ b = 80 \end{array}}$

$6(4)^2 = 2a(4-3)$

$96 = 2a$

$48 = a$

These are not true!

$\cancel{2x^3 = a(x-3)^2 + b}$

$\cancel{6x^2 = 2a(x-3)}$

b. $f(x) = \begin{cases} ax^2 + 5 & , x \le 3 \\ x^2 - 4x + b & , x > 3 \end{cases}$

$f'(x) = \begin{cases} 2ax & , x < 3 \\ 2x - 4 & , x > 3 \end{cases}$

$9a + 5 = 9 - 12 + b$

$9a + 5 = -3 + b$

$9a + 8 = b$

$9(\frac{1}{3}) + 8 = b$

$11 = b$

$\boxed{\begin{array}{l} a = \frac{1}{3} \\ b = 11 \end{array}}$

$6a = 6 - 4$

$a = \frac{2}{6}$

$a = \frac{1}{3}$

Review Set 19A

9. Line through $A(2,4)$ and $B(0,8)$ is tangent to $y = \frac{a}{(x+2)^2}$

Tangent Line : $m = \frac{8-4}{0-2} = -2 \rightarrow y = -2x + 8$

$y = a(x+2)^{-2}$

$y' = -2a(x+2)^{-3}$

For some value of x, ← The tangent intersects the curve

The slope of the curve = the slope of the tangent

$a(x+2)^{-2} = -2x+8$ and $-2a(x+2)^{-3} = -2$

$a = (-2x+8)(x+2)^2$ $\quad$ $a = (x+2)^3$

$(-2x+8)(x+2)^2 = (x+2)^3$

$(-2x+8)(x+2)^3 - (x+2)^3 = 0$

$(x+2)^2[(-2x+8) - (x+2)] = 0$

$(x+2)^2(-3x+6) = 0$

$\cancel{x = -2}$, not in the domain $\quad x = 2$ ← plug $x=2$ into tangent equation

$\hookrightarrow y = -2(2) + 8$

$y = 4$

$\hookrightarrow (2,4)$ is on the curve

$4 = \frac{a}{(2+2)^2}$

$\boxed{a = 64}$