Chapter 20 Review Solution

#1. $h(t) = -16t^2 + 120t + 10$

a. $-16t^2 + 120t + 10 = 0$

$\boxed{t \approx 7.58}$ $\quad t \approx \cancel{-0.0824}$

b. $v(t) = -32t + 120$

$v(7.58) \approx \boxed{-123 \frac{ft}{sec}}$

c. $v(t) = -32t + 120 = 0 \Rightarrow t = 3.75$ sec when Max. height.

total distance $= |h(0) - h(3.75)| + |h(3.75) - h(7.58)|$

$= |10 - 235| + |235 - 0| = \boxed{460 \text{ ft}}$

\# 2. (a) $t = 8$ $\quad d(8) \approx 5.15$ m. $\quad t = 0$ (12 am) $\Rightarrow$ $\dfrac{d(8) - d(0)}{8}$ check.

$= \dfrac{16.1 - 5.15}{8} \approx 1.37$

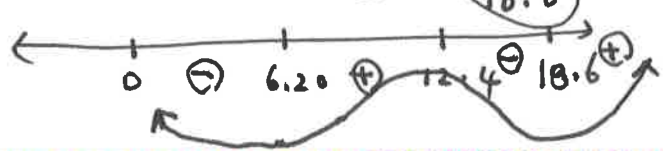
(b) $\dfrac{dd}{dt} = (-0.507)(6.8)\sin(0.507t)$

$\left.\dfrac{dd}{dt}\right|_{t=8} = (-0.507)(6.8)\sin[(0.507)(8)]$

$\approx \boxed{2.73 \frac{m}{s}}$

(c) $\dfrac{dd}{dt} = -3.4\sin(0.570t) = 0 \Rightarrow 0, \pi, 2\pi, 3\pi$

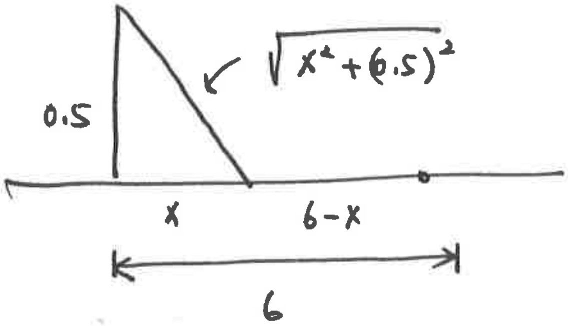
$t = 0, 6.20, 12.4, 18.6$ $\Leftarrow$ Use calculator.



Falling

$\boxed{(0, 6.2) \cup (12.4, 18.6)}$

#3.



$$\text{Cost}(x) = (8)(5280)\sqrt{x^2+(0.5)^2} + (6)(5280)(6-x)$$
(\$)

$$\left(\frac{\$8}{\cancel{\text{foot}}}\right)\left(\frac{5280\ \cancel{\text{feet}}}{1\text{ mile}}\right)$$

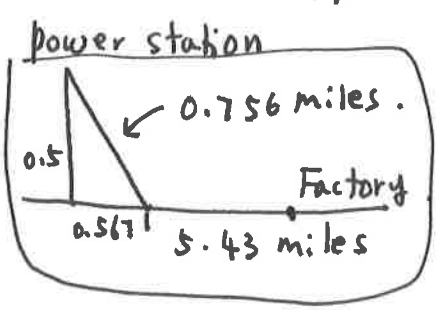
$$\frac{dc}{dx} = 0 = (8)(5280)\left(\tfrac{1}{2}\right)(2x)\left(x^2+(0.5)^2\right)^{-\frac{1}{2}} - (6)(5280)$$

$$\Rightarrow \quad \frac{8(5280)x}{\sqrt{x^2+(0.5)^2}} = 6(5280)$$

$$(4x)^2 = \left(3\sqrt{x^2+(0.5)^2}\right)^2$$

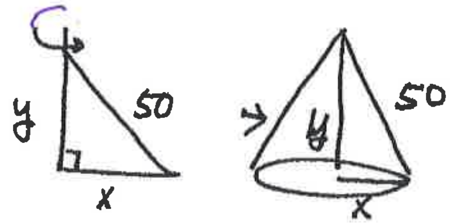
$$16x^2 = 9\left(x^2+(0.5)^2\right)$$

$$7x^2 = 9(0.5)^2 \Rightarrow x = \sqrt{\frac{9(0.5)^2}{7}} \approx \boxed{0.567\text{ miles}}$$



← path:

#4



$V = \frac{1}{3}\pi x^2 y$

$y^2 + x^2 = 50^2$

$\Rightarrow x^2 = 50^2 - y^2$

$V = \frac{1}{3}\pi y(50^2 - y^2)$

$V = ?$

$\Rightarrow V' = 0$ solve for y

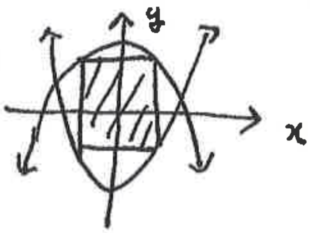
$V = \frac{1}{3}\pi[50^2 y - y^3]$

$V' = \frac{1}{3}\pi[50^2 - 3y^2] = 0$

$\Rightarrow 3y^2 = 50^2 \Rightarrow y = \frac{50}{\sqrt{3}}$ m

$\Rightarrow \boxed{V = \frac{1}{3}\pi\left(\frac{50}{\sqrt{3}}\right)\left(50^2 - \frac{50}{3}\right) \approx}$

#5.



$A = 2x \cdot y$

$y = (4 - x^2) - (x^2 - 3)$

$= -2x^2 + 7$

$\Rightarrow A = 2x(-2x^2 + 7)$

$A = -4x^3 + 14x$

$A' = 0$ solve for x

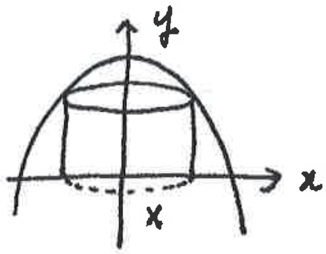
$A' = -12x^2 + 14 = 0$

$x^2 = \frac{14}{12} = \frac{7}{6}$

$x = \pm\sqrt{\frac{7}{6}}$.

$\Rightarrow \boxed{A = 2\sqrt{\frac{7}{6}}\left(-2\left(\frac{7}{6}\right) + 7\right) \approx}$

#6.



$V = \pi x^2 y, \quad y = 4 - x^2$

$V = \pi x^2(4 - x^2)$

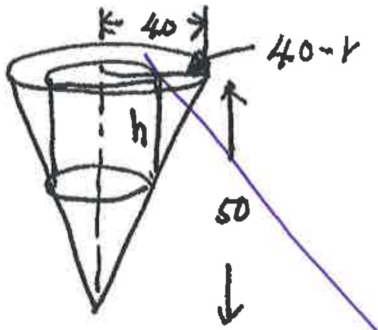
$V = 4\pi x^2 - \pi x^4$

$V = ?$

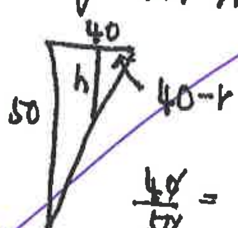
$V' = 0$ solve for x. $V' = \pi(8x - 4x^3) = 0 \Rightarrow 4x(2 - x^2) = 0$

$x = 0 \quad x = \pm\sqrt{2} \Rightarrow V = \pi \cdot 2(4 - 2) = \boxed{4\pi}$

#7.



$V = \pi r^2 h$



$\frac{40}{50} = \frac{40 - r}{h}$

$4h = 5(40 - r)$

$h = \frac{5}{4}(40 - r)$

$V = \pi r^2\left(\frac{5}{4}(40 - r)\right)$

$= \frac{5\pi}{4}(40r^2 - r^3)$

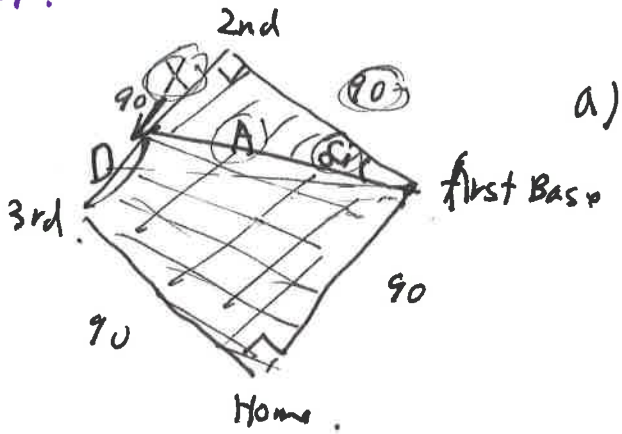
$V' = 0$

$V' = \frac{5\pi}{4}(80r - 3r^2) = 0$

$r = 0 \quad 80 - 3r = 0$

$r = \frac{80}{3}$ m, $h =$ m

#7.



a) $\frac{dA}{dt} = ?$

$A^2 = 90^2 + x^2$

$\cancel{2}A \cdot \boxed{\frac{dA}{dt}} = \cancel{2}x \cdot \frac{dx}{dt}$

$\frac{dx}{dt} = 18 \frac{ft}{sec}$. $\frac{dD}{dt} = -18 \frac{ft}{sec}$

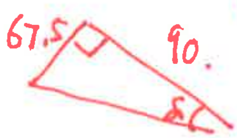
$\frac{dA}{dt} = \frac{(67.5)(18)}{112.5} \approx 10.8 \frac{ft}{sec}$

$D = 22.5$ ft.

$x = 90 - 22.5 = 67.5$ ft.

$A = \sqrt{90^2 + (67.5)^2} \approx 112.5$.

b) $\frac{d\delta}{dt} = ?$



$\tan\delta = \frac{x}{90}$

$\delta = \tan^{-1}\left(\frac{67.5}{90}\right)$

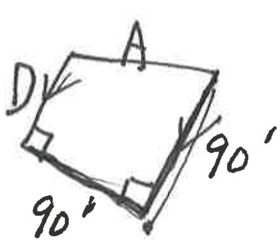
$\approx .6435$ rad

$\sec^2\delta \cdot \boxed{\frac{d\delta}{dt}} = \frac{1}{90} \cdot \frac{dx}{dt}$.

$\frac{d\delta}{dt} = \left(\frac{18}{90}\right)\left(\frac{1}{\sec^2(.6435)}\right) \approx .128 \frac{rad}{sec}$

$T = \square ABCD$.

$\frac{dT}{dt} = ?$



$\frac{dD}{dt} = -18 \frac{ft}{sec}$

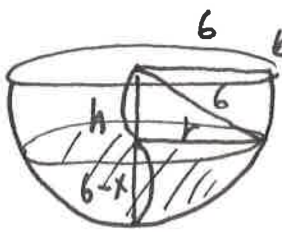
c) $T = \frac{1}{2}h(\text{Base}_1 + \text{Base}_2)$

$T = \cancel{\frac{1}{2} \cdot 90}(90 + D)$

$T = 45 \cdot 90 + 45 \cdot D$

$\frac{dT}{dt} = 45 \cdot \frac{dD}{dt} = -810 \frac{ft^2}{sec}$

#8



$\frac{dV}{dt} = 4 \frac{in^3}{sec}.$

$\Rightarrow \frac{d}{dt}(6-x)$

$\Rightarrow -\frac{d}{dt}x = ? \left(= \frac{dh}{dt}\right)$

when 2 inched depth

$h = 4$ inches

a)

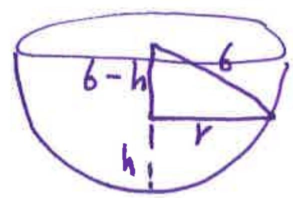
$$V = \pi h^2\left(R - \frac{h}{3}\right) = \pi h^2\left(6 - \frac{h}{3}\right)$$
$$= \pi\left[6h^2 - \frac{1}{3}h^3\right]$$

$$\frac{dV}{dt} = \pi\left(12h\frac{dh}{dt} - h^2\frac{dh}{dt}\right)$$
$$= \pi\left(12h - h^2\right)\boxed{\frac{dh}{dt}}$$

$$4 = \pi\left(12\cdot 4 - 4^2\right)\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{32\pi} = \boxed{\frac{1}{8\pi}\ \frac{in}{sec}} \text{ rising.}$$

b) $r = \sqrt{6^2 - h^2} = \sqrt{36-(6-h)^2}$



$\frac{dA}{dt} = ?$

$$\cancel{A = \pi r^2 = \pi(36-h^2)}$$

$$\cancel{\frac{dA}{dt} = \pi(-2h)\frac{dh}{dt}.}$$

$$\cancel{= \pi(-2)(4)\left(\frac{-1}{8\pi}\right)} = \boxed{1 \frac{in^2}{sec}}$$

$$A = \pi\left(36 - (6-h)^2\right)$$
$$= \pi\left(\cancel{36} - \cancel{36} + 12h - h^2\right)$$
$$= \pi\left(12h - h^2\right)$$

$$\frac{dA}{dt} = \pi(12-2h)\frac{dh}{dt} = \pi\left(12-(2)(2)\right)\left(\frac{1}{8\pi}\right) = \frac{1\ in^2}{se}$$