Chapter 22 Review Solutions

1. $v(t) = 48 - 3t^2$

The truck stops when $v(t) = 0$.

$0 = 48 - 3t^2$

$t = 4$

Stopping Distance $= \int_0^4 (48 - 3t^2)\,dt$

$= \left[48t - t^3\right]_0^4$

$= (48(4) - 4^3) - 0$

$= \boxed{128 \text{ feet}}$

2. $\dfrac{dP}{dt} = k\sqrt{t}$

$P = k \cdot \frac{2}{3} t^{\frac{3}{2}} + C$

$P(0) = 500$

$500 = \frac{2}{3}k(0)^{3/2} + C$

$500 = C$

$P(1) = 600$

$600 = \frac{2}{3}k(1)^{3/2} + 500$

$100 = \frac{2}{3}k$

$150 = k$

$P = \frac{2}{3}(150)\, t^{3/2} + 500$

$P = 100\, t^{3/2} + 500$

$P(7) = 100(7)^{3/2} + 500 \approx \boxed{2350 \text{ bacteria}}$

3. $v(t) = t^2 - 7t + 10$

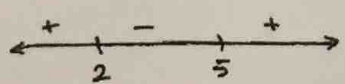
$s(t) = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 10t + C$

It doesn't matter what this is, so I'm going to use 0.

a. $s(6) - s(0)$

$\left(\frac{6^3}{3} - \frac{7 \cdot 6^2}{2} + 10 \cdot 6\right) - (0 - 0 + 0)$

$= \boxed{6 \text{ feet}}$

b. 

$v(t) = (t-2)(t-5)$ $\boxed{t = 2, 5}$

c. $s(0) = 0$

$s(2) = \frac{8}{3} - \frac{7 \cdot 4}{2} + 20 = \frac{26}{3}$ $\quad + \frac{26}{3}$

$s(5) = \frac{125}{3} - \frac{7 \cdot 25}{2} + 50 = \frac{25}{6}$ $\quad - \frac{9}{2}$

$s(6) = \frac{216}{3} - \frac{7 \cdot 36}{2} + 60 = 6$ $\quad \frac{11}{6}$

$\frac{26}{3} + \frac{9}{2} + \frac{11}{6} = \boxed{15 \text{ feet}}$

d. $a(t) = 2t - 7$

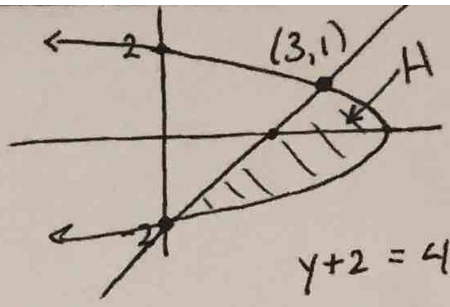
$a(4) = 8 - 7 = 1$

$v(4) = 4^2 - 7 \cdot 4 + 10 = -2$

At $t = 4$, acceleration is positive and velocity is negative, so the particle's speed is <u>decreasing</u>.

4. $x = y + 2,\quad x = 4 - y^2$

$x - 2 = y \qquad x = (2-y)(2+y)$



$y+2 = 4-y^2$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = -2, 1$

a. $\text{Area} = \int_{-2}^{1} \left((4-y^2) - (y+2)\right) dy$

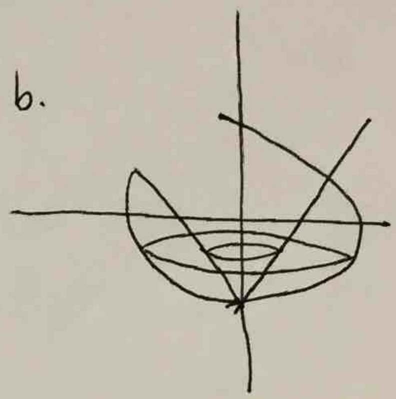
$= \int_{-2}^{1} (2 - y^2 - y)\, dy$

$= \left[2y - \frac{1}{3}y^3 - \frac{1}{2}y^2\right]_{-2}^{1}$

$= \left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 + \frac{8}{3} - 2\right) = \boxed{\frac{9}{2}}$

c. $\int_{-2}^{1} \left(\pi (4-y^2)^2 - \pi (y+2)^2\right) dy$

b.



$\pi \int_{-2}^{1} \left((16 - 8y^2 + y^4) - (y^2 + 4y + 4)\right) dy$

$\pi \int_{-2}^{1} (12 - 9y^2 + y^4 - 4y)\, dy$

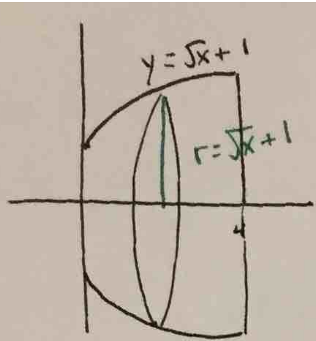
$\pi \left[12y - 3y^3 + \frac{1}{5}y^5 - 2y^2\right]_{-2}^{1}$

$\pi \left(\left(12 - 3 + \frac{1}{5} - 2\right) - \left(-24 + 24 - \frac{32}{5} - 8\right)\right)$

$\pi \left(\frac{36}{5} - \left(-\frac{72}{5}\right)\right)$

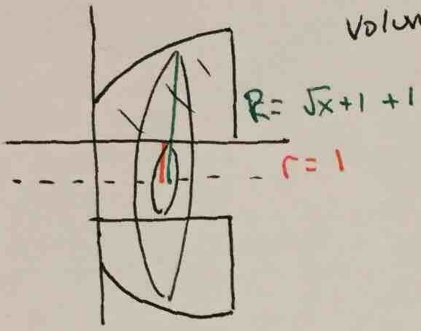
$\boxed{\frac{108\pi}{5}} = \boxed{21.6\pi}$

5. a.



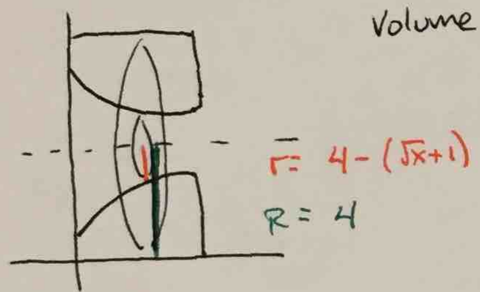
$$\text{Volume} = \int_0^4 \pi\left(\sqrt{x}+1\right)^2 dx$$

b.



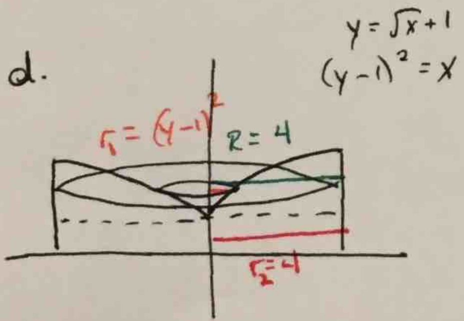
$$\text{Volume} = \pi\int_0^4 \left(\left(\sqrt{x}+2\right)^2 - 1^2\right) dx$$

c.



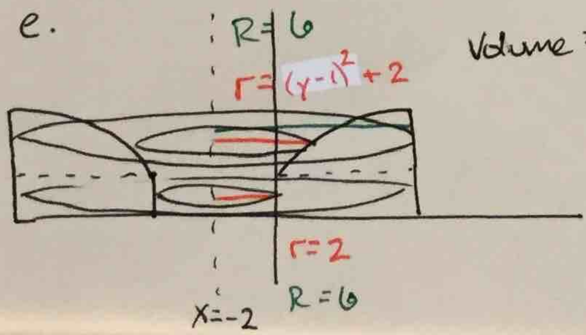
$$\text{Volume} = \pi\int_0^4 \left(4^2 - \left(4-(\sqrt{x}+1)\right)^2\right) dx$$

d.



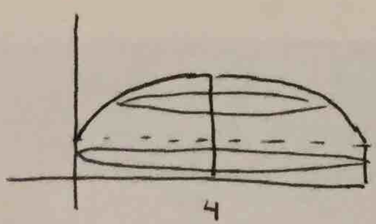
$$\text{Volume} = \int_0^1 4^2\pi\, dx + \pi\int_1^3 \left(4^2 - \left((y-1)^2\right)^2\right) dy$$

e.



$$\text{Volume} = \pi\int_0^1 \left(6^2-2^2\right) dx + \pi\int_1^3 \left(6^2 - \left((y-1)^2+2\right)^2\right) dy$$

f.



$$\text{Volume} = \pi\int_1^3 \left(4-(y-1)^2\right)^2 dy + \pi\int_0^1 4^2\, dy$$