IB Math HL2 Complex numbers

$(a+b)(a-b)$
$= a^2 - b^2$

11/29 Wednesday:

Read p 480 Ch 16 A: Complex numbers as 2-D vectors
Do the homework Ch 16 A p482 #1 b and c, #2 c, #4 f and g, #5 and #8

Read p 483 Ch B: Modulus
Do the homework Ch 16 B p 484 #2 d and g, #4, #7 b and c.

12/1 Friday:

Ch 16 C: Argument and Polar Form

Cartesian From

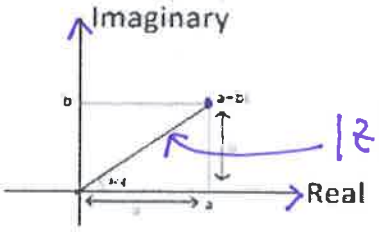
- Complex Number: $z = a + bi$
- The conjugate of z: $z^* = a - bi$
- Modulus: $|z| = \sqrt{a^2 + b^2}$

Argand Plane (Complex Plane)

$z \cdot z^* = (a+bi)(a-bi) = a^2 + b^2$

$Re(z) = a$

$Im(z) = b$



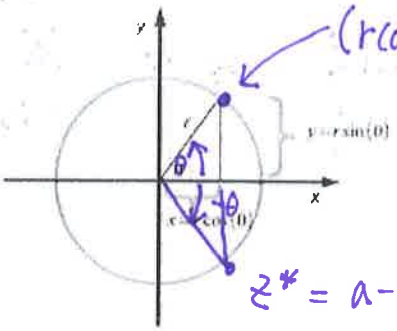
$|z| = \sqrt{a^2+b^2}$

Polar Form

- Complex Number: $z = r\cos\theta + ir\sin\theta = rcis\theta$ → $r(\cos\theta + i\sin\theta)$
- The conjugate of z: $z^* = r\cos(-\theta) + ir\sin(-\theta) = rcis(-\theta)$
- Modulus: $r = |z| = \sqrt{a^2 + b^2}$
- Argument of z is θ: $Arg(z) = \theta$ where $-\pi \le \theta \le \pi$

Argand Plane (Complex Plane)

$a + bi$
$= r\cos\theta + ir\sin\theta$.



$(r\cos\theta, r\sin\theta)$

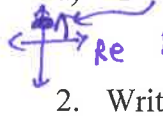
$z^* = a - bi$
$= r\cos(-\theta) + ir\sin(-\theta)$

$z = r\,cis\,\theta$

Examples) No Calculator

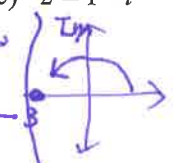
1. Write $z = a + bi$ in polar form.

a) $z = 2i$



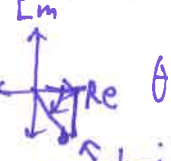
$r = |\sqrt{2^2}| = 2$. $\theta = \frac{\pi}{2}$ $z = 2\,cis(\frac{\pi}{2})$

b) $z = -3$



$\theta = \pi$. $z = 3\,cis(\pi)$
$r = 3$

c) $z = 1 - i$

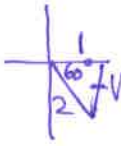


$\theta = -\frac{\pi}{4}$ $r = \sqrt{1^2+1^2} = \sqrt{2}$.

$z = \sqrt{2}\,cis(-\frac{\pi}{4})$

2. Write $z = rcis\theta$ in Cartesian form

a) $z = 3cis(-\frac{\pi}{3})$



$= 3(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3}))$

$= \frac{3}{2} - \frac{3\sqrt{3}}{2}i$

b) $z = \sqrt{5}cis(\frac{5\pi}{6})$

$= \sqrt{5}(\cos(\frac{5\pi}{6}) + \sqrt{5}i\sin(\frac{5\pi}{6}))$

$= -\frac{\sqrt{15}}{2} + \frac{\sqrt{5}}{2}i$

The Properties of $z = rcis\theta$

- $cis\theta \times cis\beta = cis(\theta + \beta)$
- $\dfrac{cis\theta}{cis\beta} = cis(\theta - \beta)$
- $cis(\theta + 2\pi k) = cis\theta$ where $k \in \mathbb{Z}$

Examples)

1. Use the properties of $z = rcis\theta$ **to** Simplify. Write the answer in Cartesian form (Surd form)

a) $cis(\frac{\pi}{5})cis(\frac{3\pi}{10}) = Cis(\frac{\pi}{5} + \frac{3\pi}{10})$

$= Cis(\frac{\pi}{2}) = \cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2})$

$= i$

b) $\dfrac{cis(\frac{\pi}{5})}{cis(\frac{7\pi}{10})} = Cis(\frac{\pi}{5} - \frac{7\pi}{10})$

$= Cis(-\frac{\pi}{2}) = \cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2})$

$= -i$

2. Given $z = \sqrt{2}cis\theta$, find the modulus and argument of

$(1-i) = \sqrt{2}Cis(-\frac{\pi}{4})$

a) $2z$

$2z = 2\sqrt{2}Cis\theta$

$|2z| = 2\sqrt{2}$

$Arg(z) = \theta$

b) $\frac{z}{i}$

$i = Cis(\frac{\pi}{2})$

$\Rightarrow \dfrac{z}{i} = \dfrac{\sqrt{2}Cis\theta}{Cis(\frac{\pi}{2})} = \sqrt{2}Cis(\theta - \frac{\pi}{2})$

$|\frac{z}{i}| = \sqrt{2}$

C) $(1-i)z$

$= \sqrt{2}Cis(-\frac{\pi}{4}) \cdot \sqrt{2}Cis(\theta) = 2Cis(\theta - \frac{\pi}{4})$

$|(1-i)z| = 2$

$Arg(z) = \theta - \frac{\pi}{4}$

$Arg(z) = \theta - \frac{\pi}{2}$

- *Notes: Remember Argument of z is θ: $Arg(z) = \theta$ where $-\pi \le \theta \le \pi$*