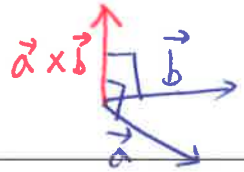
14K The Vector Product of Two Vectors (Cross Product)-Day 1

The **vector cross product** of vectors	
$\mathbf{a}=\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$ is $\mathbf{a}\times\mathbf{b}=\begin{pmatrix}a_2b_3-a_3b_2\\a_3b_1-a_1b_3\\a_1b_2-a_2b_1\end{pmatrix}$	Tip: Use the determinant! $\mathbf{a}\times\mathbf{b}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\a_1&a_2&a_3\\b_1&b_2&b_3\end{vmatrix}=i(a_2b_3-a_3b_2)-j(a_1b_3-a_3b_1)+k(a_1b_{\cancel{3}2}-a_2b_1)$ Remember +, - , +



Examples: Use $\mathbf{a}=\begin{pmatrix}2\\1\\-1\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}1\\2\\3\end{pmatrix}$, and $\mathbf{c}=\begin{pmatrix}2\\0\\4\end{pmatrix}$ to evaluate the expressions.

1. $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & 2 & 3 \end{vmatrix} =$

$= i(1\cdot3-(-1\cdot2))-j(2\cdot3-(-1\cdot1))+k(2\cdot2-1\cdot1)$

$= \boxed{5i-7j+3k}$

2. $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{a} = \begin{pmatrix}5\\-7\\3\end{pmatrix}\cdot\begin{pmatrix}2\\1\\-1\end{pmatrix}$

$= 10-7-3 = \boxed{0}$

3. $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{b} = \begin{pmatrix}5\\-7\\3\end{pmatrix}\cdot\begin{pmatrix}1\\2\\3\end{pmatrix}$

$= 5-14+9 = \boxed{0}$

4. $\mathbf{a}\times\mathbf{a} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 2 & 1 & -1 \end{vmatrix}$

$= i(-1+1)-j(-2+2)+k(2-2)$

$= \boxed{0}$

5. $-\mathbf{b}\times\mathbf{a} = \begin{vmatrix} i & j & k \\ -1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix}$ $= i(5)-j(1+6)+k(3)$

$= \boxed{5i-7j+3k}$

6. $\mathbf{a}\times(\mathbf{b}+\mathbf{c})$

$b+c = \begin{pmatrix}1\\2\\3\end{pmatrix}+\begin{pmatrix}2\\0\\4\end{pmatrix}=\begin{pmatrix}3\\2\\7\end{pmatrix}$

$a\times(b+c) = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 3 & 2 & 7 \end{vmatrix}$

$= \boxed{9i-17j+k}$

7. ~~$\mathbf{a}\times\mathbf{b}$~~ $b\times a$

$= \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 2 & 1 & -1 \end{vmatrix}$

$= \boxed{-5i+7j-3k}$

8. $\mathbf{a}\times\mathbf{c} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 2 & 0 & 4 \end{vmatrix}$

$= \boxed{4i-10j-2k}$

Algebraic Properties of the Vector Cross Product Use the results of problems 1 – 8 to fill in the blanks.

$\mathbf{a}\times\mathbf{b}$ is a Vector which is perpendicular to both **a** and **b**. (#1 – 3)

$\mathbf{a}\times\mathbf{a} =$ 0 for all **a**.

$a \times b = -\mathbf{b}\times\mathbf{a}$ for all **a** and **b**.

Hence $\mathbf{a}\times\mathbf{b}$ and $\mathbf{b}\times\mathbf{a}$ have the Same length and opposite direction.

$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})$ is called the **scalar triple product**.

$\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = (a \times b) + (a \times c)$ (#6 – 8)

$(\mathbf{a}+\mathbf{b})\times(\mathbf{c}+\mathbf{d}) = (\mathbf{a}\times c) + (\mathbf{a}\times d) + (\mathbf{b}\times c) + (\mathbf{b}\times d)$

Example 9) $a = 2i - 5j + k$ and $b = -i + 3j + 6k$.

a. Find two perpendicular vectors to both $a = 2i - 5j + k$ and $b = -i + 3j + 6k$.

$$a \times b = \begin{vmatrix} i & j & k \\ 2 & -5 & 1 \\ -1 & 3 & 6 \end{vmatrix} = \boxed{-33i - 13j + k}$$

$$b \times a = \boxed{33i + 13j - k}$$

b. Verify your answers of a are perpendicular to a and b.

$$(a \times b) \cdot a = \begin{pmatrix} -33 \\ -13 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix} = -66 + 65 + 1 = \boxed{0} \quad \therefore (a \times b) \perp a$$

$$(a \times b) \cdot b = \begin{pmatrix} -33 \\ -13 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ 6 \end{pmatrix} = 33 - 39 + 6 = \boxed{0} \quad \therefore (a \times b) \perp b.$$