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Use this if a student cannot read

IB Math HL1 (Cumulative Review IB Questions)-No Calculator

Work as much as you can with your group members only and turn in at the end of period. This is not homework.

Names: key Period: ______

I will post the answers and Questions after the Break.

1 a Find $\sum_{r=1}^{3}(2r+2^r)$.

b Find an expression for $\sum_{r=1}^{n}(2r+2^r)$.

a) 26

b) $\frac{n}{2}(2+2n)+\frac{2(2^n-1)}{2-1}$

2 If $\log_a 2 = b$ and $\log_a 3 = c$, express $\log_a\sqrt{72}$ in terms of b and c.

$\log_a\sqrt{72} = \log_a(2^3\cdot 3^2)^{\frac{1}{2}}$

$= \frac{1}{2}\cdot 3\log_a 2 + \frac{1}{2}\cdot 2\cdot\log_a 3 = \boxed{\frac{3}{2}b+c}$

3 Find the exact value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}(\cos^2 x+\tan^2 x)\,dx$.

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left[\frac{1}{2}+\frac{1}{2}\cos(2x)+\sec^2 x-1\right]dx = \left[\frac{1}{4}\sin 2x+\tan x-\frac{1}{2}x\right]_{x=\frac{\pi}{6}}^{x=\frac{\pi}{3}}$

$= \left[\frac{1}{4}\left(\frac{\sqrt{3}}{2}\right)+\sqrt{3}-\frac{\pi}{6}\right] - \left[\frac{1}{4}\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{3}}-\frac{\pi}{12}\right] = \boxed{\sqrt{3}-\frac{1}{\sqrt{3}}-\frac{\pi}{12}}$

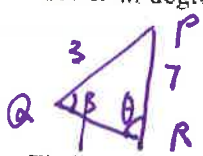
4 Factorise $f(x) = 2x^3 - x^2 - 8x - 5$ into real factors, and hence determine the values of x for which $f(x) > 0$.

$f(x) = (x+1)(2x^2-3x-5) = (x+1)(x+1)(2x-5)$

$x > \frac{5}{2}$ $f(x) > 0$

Sign diagram for $f(x)$.

5 In triangle PQR, $\sin P\hat{R}Q = \frac{3\sqrt{3}}{14}$, $QP = 3$ cm, and $PR = 7$ cm. Find the possible values of $P\hat{Q}R$, giving your answer in degrees.



$\sin\theta = \sin P\hat{R}Q = \frac{3\sqrt{3}}{14}$

$\Rightarrow \frac{\sin\theta}{3} = \frac{\sin\beta}{7}$

$\frac{\frac{3\sqrt{3}}{14}}{3} = \frac{\sin\beta}{7} \Rightarrow \sin\beta = \frac{\sqrt{3}}{2}$

$\boxed{\beta = 60° \text{ OR } 120°}$

6 Find the exact value of $\sin\theta$ if $3\cos 2\theta + 2 = 7\sin\theta$.

$3(1-2\sin^2\theta)+2-7\sin\theta = 0$

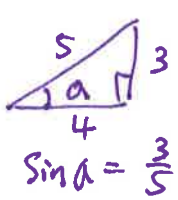
$6\sin^2\theta+7\sin\theta-5 = 0$

$(2\sin\theta-1)(3\sin\theta+5) = 0$

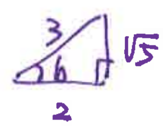
$\Rightarrow \sin\theta = \frac{1}{2}$ $\cancel{\sin\theta = \frac{-5}{3}}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ for $0 \le \theta \le 2\pi$

7 Find the exact value of $\cos\left(\arcsin\frac{3}{5}+\arccos\frac{2}{3}\right)$.



$\sin a = \frac{3}{5}$



$\cos b = \frac{2}{3}$

$\Rightarrow \cos(a+b)$

$= \cos a\cdot\cos b - \sin a\cdot\sin b$ ⇐ Compound Angle identities.

$= \frac{4}{5}\times\frac{2}{3} - \frac{3}{5}\times\frac{\sqrt{5}}{3} = \boxed{\frac{8-3\sqrt{5}}{15}}$

8 Consider the points $X(3\mu, 2, 1)$ and $Y(\mu, 1-3\mu, 2\mu-1)$, where μ is a constant.

If O is the origin, find all values of μ for which $\overrightarrow{OX}$ is perpendicular to $\overrightarrow{OY}$.

Dot product $=0$ for $\perp$ vectors.

$$\begin{pmatrix} 3\mu \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} \mu \\ 1-3\mu \\ 2\mu-1 \end{pmatrix} \Rightarrow 3\mu^2 + 2(1-3\mu) + (2\mu-1) = 0$$

$$3\mu^2 - 4\mu + 1 = 0 \quad (3\mu-1)(\mu-1) = 0 \quad \Rightarrow \boxed{\mu = \tfrac{1}{3} \text{ or } \mu = 1}$$

9 If the polynomial $x^n + ax^2 - 6$ leaves a remainder of -3 when divided by $(x-1)$, and a remainder of -15 when divided by $(x+3)$, find the values of a and n.

$$p(1) = -3 \Rightarrow 1 + a - 6 = -3 \quad \therefore \boxed{a = 2}$$

$$p(-3) = -15 \Rightarrow (-3)^n + 2(-3)^2 - 6 = -15 \Rightarrow (-3)^n = -27$$

$$\boxed{n = 3}$$

10 Consider the function $f : x \mapsto a + \dfrac{b}{x+c}$.

The graph of f has asymptotes $x = -2$ and $y = 3$, and passes through $(2, 4)$.

a Find the values of a, b, and c.

b State the domain and range of f.

c Find f^{-1} and state the domain and range of f^{-1}.

a) $\boxed{a = 3, \quad b = 4, \quad c = 2}$

b) Domain of $f(x)$ $x \in \mathbb{R}$ $x \neq -2$
Range of $f(x)$ $y \in \mathbb{R}$ $y \neq 3$

c) $\boxed{f^{-1}(x) = \dfrac{4}{x-3} - 2}$

Domain of $f^{-1}(x)$: $x \in \mathbb{R}$ $x \neq 3$
Range of $f^{-1}(x)$: $y \in \mathbb{R}$ $y \neq -2$

11 Find k if $\displaystyle\int_1^k 3\sqrt{10-x}\,dx = 38$.

$$-3 \cdot \tfrac{2}{3} (10-x)^{3/2} \Big|_{x=1}^{x=k} \Rightarrow (10-k)^{3/2} - 27 = -19$$

$$\boxed{k = 6}$$

12 **a** Prove that $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$.

Out $\Rightarrow$ **b** ~~Hence, or otherwise, find the exact value of the period of the function $f(x) = \sin 5x \cos 2x$.~~

c Find $\int \sin 5x \cos 2x \, dx$.

d Hence find the exact value of $\int_0^{\frac{\pi}{4}} \sin 5x \cos 2x \, dx$.

~~**a** Prove that $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$.~~

~~**b** Hence, or otherwise, find the exact value of the period of the function $f(x) = \sin 5x \cos 2x$.~~

~~**c** Find $\int \sin 5x \cos 2x \, dx$.~~

~~**d** Hence find the exact value of $\int_0^{\frac{\pi}{4}} \sin 5x \cos 2x \, dx$.~~

#12 Solution attached

#12 .

a) $\sin(A+B) = \sin A \cdot \sin B + \cos A \cos B$

$+\ \sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$

b) $f(x) = \sin 5x \cos 2x = \frac{1}{2}\left[2 \sin 5x \cos 2x\right]$

$= \frac{1}{2}\left[\ \sin 7x + \sin 3x\right]$

$\sin 7x \Rightarrow \frac{6\pi}{21}, \frac{12\pi}{21}, \frac{18\pi}{21}, \frac{42\pi}{21} \ldots$

$\sin 3x \Rightarrow \frac{14\pi}{21}, \frac{28\pi}{21}, \ldots$

LCM of 6 and 14 $\Rightarrow 42 \Rightarrow \frac{42\pi}{21} \Rightarrow \boxed{2\pi}$

We did not cover this material.

c) $\int (\sin 5x \cos 2x)\, dx = \int \frac{1}{2}\left[\sin 7x + \sin 3x\right] dx = -\frac{1}{14} \cos 7x - \frac{1}{6} \cos 3x + C$

d)

$\int_0^{\frac{\pi}{3}} \frac{1}{2}(\sin 7x + \sin 3x)dx = \frac{1}{2}\left[-\frac{1}{7} \cos 7x - \frac{1}{3} \cos 3x\right]_{x=0}^{x=\frac{\pi}{3}}$

$= \frac{1}{2}\left[-\frac{1}{7} \cos\left(\frac{7\pi}{3}\right) - \frac{1}{3} \cos\left(\frac{3\pi}{3}\right)\right] + \frac{1}{2}\left[\frac{1}{7} \cos 0 + \frac{1}{3} \cos 0\right]$

$= -\frac{1}{28} + \frac{1}{6} + \frac{1}{14} + \frac{1}{6} = \boxed{\frac{31}{84}}$