IB Math HL1: Curve Analysis Practice: Work with your group members
Use of Calculator is okay when appropriate. However, must show your work.

Names: key

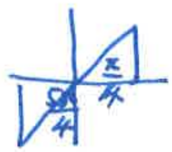
1) Given: $f(x) = \sin x + \cos x \quad x \in [0, 2\pi]$.

(Identities: $\sin 2x = 2\sin x \cos x$, $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$)

a) Find the stationary points expressed in ordered pair (x, y). $\left(\frac{\pi}{4}, \sqrt{2}\right)$ $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$
Work:

$$\frac{df}{dx} = \cos x - \sin x = 0$$

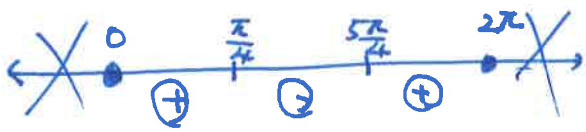
$$\cos x = \sin x$$

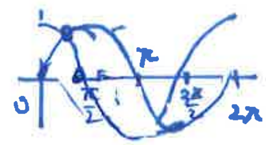


$x = \frac{\pi}{4}$ $\quad y = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$

$x = \frac{5\pi}{4}$ $\quad y = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$

b) Find the intervals where f(x) is increasing and decreasing.
Increasing: $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$ Decreasing: $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$
Justify by sign diagram:



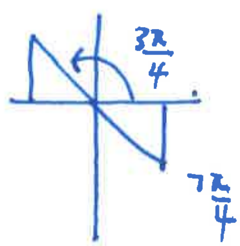


$f'(x) = \cos x - \sin x$

c) Find the inflection point(s) just for x values. $x = \frac{3\pi}{4}, \frac{7\pi}{4}$.
Work:

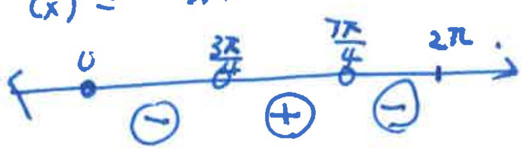
$$\frac{d^2f}{dx^2} = -\sin x - \cos x = 0$$

$$-\sin x = \cos x$$



d) Find the intervals where f(x) is concave up and concave down.
Concave up: $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$ Concave down: $\left[0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right]$
Justify by sign diagram:

$f''(x) = -\sin x - \cos x$



e) State the local max or min and justify. . Max: $x = \frac{\pi}{4}$ Min: $x = \frac{5\pi}{4}$

First Derivative test:

$f'(0) > 0$ $\quad f'\left(\frac{\pi}{2}\right) < 0$ $\quad f'\left(\frac{\pi}{2}\right) < 0$ $\quad f'(\pi) > 0$.

key ②

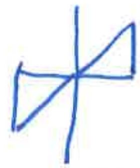
2) For the given $f(x)=e^{-x}\sin x$ on $x\in[0,\ 2\pi]$

e) Find the y-intercept and the x-intercepts.
y-int: $(0,0)$ x-int: $(0,0)$

$f(0)=e^{0}\sin 0=0$ $0=e^{-x}\sin x$

f) Find the stationary points expressed in ordered pair (x, y). $\left(\frac{\pi}{4},\ \frac{\sqrt{2}}{2e^{\frac{\pi}{4}}}\right)$
Work:

$$\frac{df}{dx}=-e^{-x}\sin x+e^{-x}\cos x$$

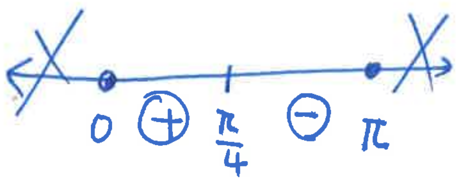


$$=e^{-x}[\cos x-\sin x]=0$$

$e^{-x}\neq 0$ $\cos x=\sin x \Rightarrow x=\frac{\pi}{4}$

$$y=e^{-\frac{\pi}{4}}\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}\cdot e^{-\frac{\pi}{4}}$$

g) Find the intervals where f(x) is increasing and decreasing. [2 pts]
Increasing: $[0,\ \frac{\pi}{4})$ Decreasing: $(\frac{\pi}{4},\ \pi]$
Justify by sign diagram:



: Sign diagram of $f'(x)$

h) Find the inflection point(s) just for x values. ________[4 pts]
Work:

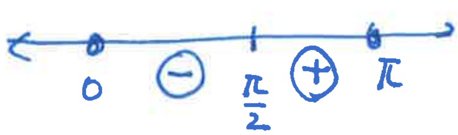
$$\frac{df}{dx}=e^{-x}[\cos x-\sin x]$$

$$\frac{d^2f}{dx^2}=-e^{-x}[\cos x-\sin x]+e^{-x}[-\sin x-\cos x]=0$$

$$=-e^{-x}\cos x+\cancel{e^{-x}\sin x}-\cancel{e^{-x}\sin x}-e^{-x}\cos x=-2e^{-x}\cos x=0$$

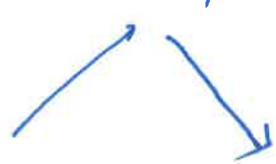
$\cos x=0$ $x=\frac{\pi}{2},$

i) Find the intervals where f(x) is concave up and concave down. [3 pts]
Concave up: $(\frac{\pi}{2},\ \pi]$ Concave down: $[0,\ \frac{\pi}{2})$
Justify by sign diagram:



: Sign diagram of $f''(x)$

j) State the local max or min and justify. . Max: $x=\frac{\pi}{4}$ Min: None



key ③

3) Given $f(x)=\dfrac{x^2-1}{x^2+1}$

4) Find the stationary points expressed in ordered pair (x, y). $(0,-1)$

Work:

$$\frac{df}{dx}=\frac{2x(x^2+1)-(x^2-1)(2x)}{(x^2+1)^2}=\frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}=\frac{4x}{(x^2+1)^2}=0$$

$x=0,$

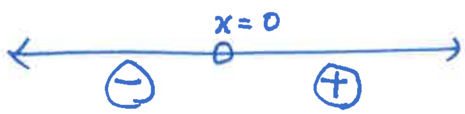
$y=\frac{0-1}{0+1}=-1$

5) Find the intervals where f(x) is increasing and decreasing.

Increasing: $(0,\infty)$ Decreasing: $(-\infty,0)$

Justify by sign diagram:

Sign diagram of $f'(x)=\dfrac{4x}{(x^2+1)^2}$



6) Find the inflection point(s) just for x values. $x=\pm\sqrt{\frac{1}{3}}$ OR $\left(\pm\frac{\sqrt{3}}{3}\right)$

Work:

$$\frac{d^2f}{dx^2}=\frac{4(x^2+1)^2-(4x)(2)(x^2+1)(2x)}{(x^2+1)^4}=\frac{4(x^4+2x^2+1)-16x^2(x^2+1)}{(x^2+1)^4}$$

$$=\frac{4x^4+8x^2+4-16x^4-16x^2}{(x^2+1)^4}=\frac{-12x^4-8x^2+4}{(x^2+1)^4}=\frac{-4(3x^4+2x^2-1)}{(x^2+1)^4}$$

7) Find the intervals where f(x) is concave up and concave down.

Concave up: $\left(-\sqrt{\frac{1}{3}},\sqrt{\frac{1}{3}}\right)$ Concave down: $\left(-\infty,-\sqrt{\frac{1}{3}}\right)\cup\left(\sqrt{\frac{1}{3}},\infty\right)$

Justify by sign diagram:

Sign diagram of $f''(x)=\dfrac{-4(3x^2-1)(x^2+1)}{(x^2+1)^4}$

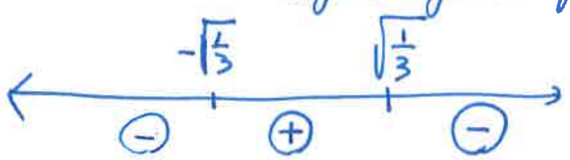
$\Rightarrow -4(3x^4+2x^2-1)=0$

$3x^2 \quad -1$

$x^2 \quad +1$

$(3x^2-1)(x^2+1)=0$

$x^2=\frac{1}{3} \quad x=\pm\sqrt{\frac{1}{3}}$



e) State the local max or min and justify. Max: ______ Min: $x=0$

by first derivative sign diagram.