IB Math HL1 : Curve Analysis IB Questions

Period: key.

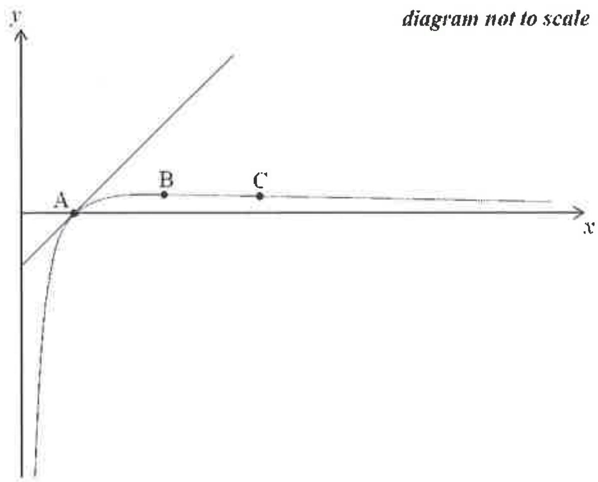
Name: ______________

NO CALCULATORS

All work must be shown for full credit. Insufficient support

Consider the function $f(x)=\dfrac{\ln x}{x},\ x>0$.

The sketch at right shows the graph of $y=f(x)$ and its tangent at point A.



a) Find the y-intercept and the x-intercepts. [1 pts]

y-int: None x-int: (1, 0)

$\ln x = 0 \Rightarrow x = 1$

b) Find the equation of tangent at point A. $y = x-1$ [2 pts]

$f'(x)=\dfrac{\left(\frac{1}{x}\right)\cdot x-\ln x}{x^2}=\dfrac{1-\ln x}{x^2}=\dfrac{1-0}{1^2}=1 \Rightarrow y=x-1$

c) Find the coordinates of B, critical point by finding $f'(x)$. [2 pts]

$f'(x)=0$.

$f'(x)=\dfrac{1-\ln x}{x^2}=0 \Rightarrow \ln x=1$

$x=e$

$\boxed{\left(e,\ \frac{1}{e}\right)}$

d) Find the intervals where f(x) is increasing and decreasing. [2 pts]

Increasing: $(0,\ e)$ Decreasing: $(e,\ \infty)$

Justify by the sign diagram of $f'(x)$:

Sign diagram of $f'(x)$

$\oplus$ $\ominus$

e

e) Find the inflections point of C. $\left(e^{\frac{3}{2}},\ \frac{3}{2}e^{-\frac{3}{2}}\right)$ [2 pts]

$f''(x)=\dfrac{-\frac{1}{x}\cdot x^2-2x(1-\ln x)}{x^4}=0 \Rightarrow -x-2x(1-\ln x)=0$

$-x\left[1+2(1-\ln x)\right]=0$

f) Find the intervals where f(x) is concave up and concave down. [2 pts]

$x=0$ $3-2\ln x=0$

$\ln x=\frac{3}{2}$

$x=e^{\frac{3}{2}}$.

Concave up: $\left(e^{\frac{3}{2}},\ \infty\right)$ Concave down: $\left(0,\ e^{\frac{3}{2}}\right)$

Justify by the sign diagram of $f''(x)$.

$f\left(e^{\frac{3}{2}}\right)=\dfrac{\ln e^{\frac{3}{2}}}{e^{\frac{3}{2}}}=\dfrac{\frac{3}{2}}{e^{\frac{3}{2}}}$.

$\ominus$ $\oplus$

$e^{\frac{3}{2}}$

$f''(x)=\dfrac{-x\left[3-2\ln x\right]}{x^4}$

2. Let $f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{4}}$

c) Find the y-intercept and the x-intercepts. [1 pts]
y-int: $\left(0, \frac{1}{\sqrt{2\pi}}\right)$ x-int: None .

$x=0$

d) Find the stationary points expressed in ordered pair (x, y). $0, \frac{1}{\sqrt{2\pi}}$. [2 pts]

$$f'(x) = \frac{1}{\sqrt{2\pi}} \cdot \left(\frac{1}{\cancel{4}}\right)(-\cancel{2}x) \cdot e^{-\frac{x^2}{4}} = 0$$

$x = 0.$

$y = \frac{1}{\sqrt{2\pi}}$

e) Find the intervals where f(x) is increasing and decreasing. [2 pts]
Increasing: $(-\infty, 0)$ Decreasing: $(0, \infty)$

$f'(x)$ Sign diagram: $\boxed{f'(x) = \frac{-\cancel{2}x}{2\cancel{4}\sqrt{2\pi}\, e^{\frac{x^2}{4}}}} = \frac{-1}{2\sqrt{2\pi}} x \cdot e^{-\frac{x^2}{4}}$

f) Find the inflections point(s) just for x values. ± 1 [2 pts]

$$f''(x) = \frac{-1}{2\sqrt{2\pi}} \left[e^{-\frac{x^2}{4}} + x\left(-\frac{2x}{4}\right) \cdot e^{-\frac{x^2}{4}} \right]$$

$$= \frac{-1}{2\sqrt{2\pi}} \cdot e^{-\frac{x^2}{4}} \left[1 - x^2\right] = 0 \Rightarrow x = \pm 1$$

g) Find the intervals where f(x) is concave up and concave down. [2 pts]
Concave up: $(-\infty, -1) \cup (1, \infty)$ Concave down: $(-1, 1)$

Sign diagram $f''(x)$