Day Two : Solutions.

#1. (a) $\log_2(x-5)+\log_2(x+2)=3$

$\Rightarrow \log_2(x-5)(x+2)=3$

$\Rightarrow (x-5)(x+2)=2^3$

$\Rightarrow x^2-3x-18=0$

$\Rightarrow (x-6)(x+3)=0$

$\boxed{x=6}$, $x\neq -3$

(b) $2\sin 2x=\sqrt{3}$

$\Rightarrow \sin 2x=\frac{\sqrt{3}}{2}$.

$0\le x\le \pi \Rightarrow \boxed{0\le 2x\le 2\pi}$

$2x=\frac{\pi}{3}$, $2x=\frac{2\pi}{3}$

$\Rightarrow \boxed{x=\frac{\pi}{6}, \ x=\frac{\pi}{3}}$

$2x=\frac{7\pi}{3} \Rightarrow x=\frac{7\pi}{6}$.

(c) $\log_{\frac{1}{2}} x=\log_2 x-1$

$\Rightarrow 1=\log_2 x-\log_{\frac{1}{2}} x$

$\Rightarrow 1=\log_2 x-\dfrac{\log_2 x}{\log_2 \frac{1}{2}}$

$\Rightarrow 1=\log_2 x-\dfrac{\log_2 x}{-\log_2 2}$

$\Rightarrow 1=\log_2 x+\log_2 x$

$\Rightarrow 1=\log_2 x^2$

$\Rightarrow x^2=2 \Rightarrow x=\pm\sqrt{2} \Rightarrow \boxed{x=\sqrt{2}}$

#2 (a)

$$\frac{1}{4}+\frac{1}{V}=\frac{10}{w}$$

$$\Rightarrow \frac{1}{2+3i}+\frac{1}{3+2i}=\frac{10}{w}$$

$$\Rightarrow \frac{(2-3i)}{(2+3i)(2-3i)}+\frac{3-2i}{(3+2i)(3-2i)}=\frac{10}{w}$$

$$\Rightarrow \frac{2-3i}{4+9}+\frac{3-2i}{9+4}=\frac{10}{w}$$

$$\Rightarrow \frac{5-5i}{13}=\frac{10}{w}$$

$$\Rightarrow \frac{5(1-i)}{13}=\frac{10}{w}\ 2$$

$$\Rightarrow \frac{1-i}{13}=\frac{2}{w}$$

$$\Rightarrow w(1-i)=26$$

$$\Rightarrow w=\frac{26(1+i)}{(1-i)(1+i)}$$

$$w=\frac{26^{13}(1+i)}{2}$$

$$\boxed{w=13+13i}$$

(c) r is a common Ratio.

$$\alpha=\frac{\beta}{r},\ \beta,\ \gamma=\beta\cdot r$$

$$\Rightarrow ① \ 6=\frac{\beta}{r}+\beta+\beta\cdot r=\beta\left(\frac{1}{r}+1+r\right)$$

$$② \ 18=\left(\frac{\beta}{r}\cdot\beta\right)+\left(\beta\cdot r\cdot\beta\right)+\left(\beta\cdot\frac{\beta}{r}\right)$$

$$18=\beta^2\left(\frac{1}{r}+1+r\right)$$

$$\Rightarrow \frac{18}{6}=\boxed{3=\beta} \Rightarrow C=-\frac{\beta}{r}\cdot\beta\cdot\beta\cdot r=-(\beta)^3$$

$$C=-27$$

#3.

(a) (i) Expand

$$(x-\alpha)(y-\beta)(x-\gamma)$$

$$=x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha\beta\gamma$$

$$=x^3+px^2+qx+c$$

$$\Rightarrow \therefore p=-(\alpha+\beta+\gamma)$$

$$q=(\alpha\beta+\beta\gamma+\gamma\cdot\alpha)$$

$$c=-\alpha\beta\gamma$$

(b) $p=-6$ and $q=18$

$$\Rightarrow \alpha+\beta+\gamma=6$$

$$\Rightarrow (\beta-\alpha)+(\beta)+(\beta+\alpha)=6$$

$$3\beta=6 \Rightarrow \boxed{\beta=2}$$

$$6=\alpha+2+\gamma$$

$$\Rightarrow \boxed{\alpha+\gamma=4} \Rightarrow \gamma=4-\alpha$$

$$q=18=\alpha\beta+\beta\gamma+\gamma\alpha$$

$$\Rightarrow 18=(\alpha)(2)+(2)(\gamma)+\gamma\cdot\alpha$$

$$18=(\alpha)(2)+2(4-\alpha)+(4-\alpha)\cdot\alpha$$

$$\Rightarrow \alpha^2-4\alpha+10=0$$

$$\Rightarrow \alpha=\frac{4\pm\sqrt{-24}}{2}=2\pm i\sqrt{6}$$

$$\alpha=2-i\sqrt{6},\ \beta=2,\ \gamma=2+i\sqrt{6}.$$

$$\therefore C=-(2-i\sqrt{6})\cdot 2\cdot(2+i\sqrt{6})$$

$$=\boxed{-20}$$

Name: Key.

[Maximum mark: 17]

The function f is defined by $f(x) = \frac{3x}{x-2}$, $x \in \mathbb{R}$, $x \neq 2$.

(a) Sketch the graph of $y = f(x)$, indicating clearly any asymptotes and points of intersection with the x and y axes. [4]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Find all values of x for which $f(x) = f^{-1}(x)$. [3]

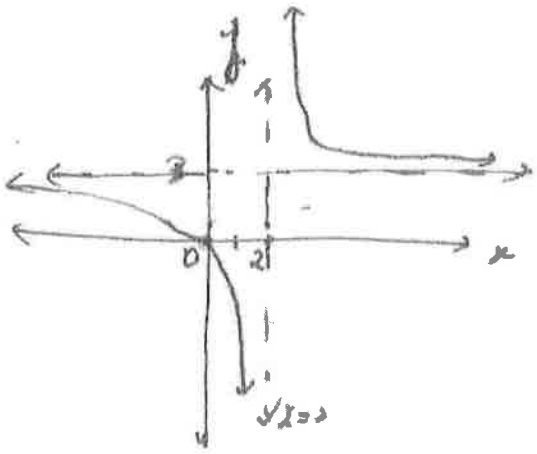
(d) Solve the inequality $|f(x)| < \frac{3}{2}$. [4]

a) V. Asymptote: $x = 2$

H Asymptote: $y = 3$

x-int ($y=0$) $\quad 0 = 3x \quad x = 0 \Rightarrow (0,0)$

y-int ($x=0$) $\quad y = 0 \Rightarrow (0,0)$



b) $x = \frac{3y}{y-2} \Rightarrow (y-2)x = 3y$

$\Rightarrow x \cdot y - 2x = 3y$

$\Rightarrow 3y - xy = -2x$

$\Rightarrow y(3-x) = -2x$

$\Rightarrow \boxed{y = \frac{2x}{x-3}} \quad (x \neq 3)$

#4.

(c) $\frac{3x}{x-2} = \frac{2x}{x-3}$

$\Rightarrow 3x(x-3) = 2x(x-2)$

$\Rightarrow 3x^2 - 9x = 2x^2 - 4x$

$\Rightarrow x^2 - 5x = 0$

$\boxed{x = 0,\ x = 5}$

#5.

(a) $\log_2(x-5) + \log_2(x+2) = 3$

$\Rightarrow \log_2(x-5)(x+2) = 3$

$\Rightarrow x^2 - 3x - 10 = 2^3$

$\Rightarrow x^2 - 3x - 18 = 0$

$+3 \quad -6$

$\Rightarrow (x+3)(x-6) = 0$

~~$x = -3$~~, $\boxed{x = 6}$

(b) $\ln(x-1) + \ln(x+1) = 4\ln 2$

$\Rightarrow \ln(x-1)(x+1) = \ln 2^4$

$\Rightarrow x^2 - 1 = 16$

$x^2 = 17$

$x = \pm\sqrt{17} \Rightarrow \boxed{x = \sqrt{17}}$