#1. a) $\frac{dy}{dx} = 4y^2 \Rightarrow \int \frac{dy}{y^2} = \int 4\,dx$

$-\frac{1}{y} = 4x + C \Rightarrow -\frac{1}{2} = 4 + C$ $(1, 2)$

$C = -\frac{1}{2} - 4 = \left[-\frac{9}{2}\right]$

$\Rightarrow \boxed{-\frac{1}{y} = 4x - \frac{9}{2}}$.

OR. $-\frac{1}{y} = \frac{8x - 9}{2} \Rightarrow \boxed{y = \frac{2}{9 - 8x}}$

#1 b). $\frac{dy}{dx} = \frac{\sin x}{y e^y}$

$y e^y \, dy = \sin x \, dx$

		dv
y	e^y	
1	e^y	
0	e^y	

$\Rightarrow y e^y - e^y = -\cos x + C$ $(0, 0)$

$-1 = -1 + C$ $\quad C = 0$

$y e^y - e^y = -\cos x$

$\boxed{e^y (y - 1) = -\cos x}$

#2. $\frac{dy}{dt} = ky \Rightarrow \frac{dy}{y} = k\,dt \Rightarrow \ln y = kt + C$

$y = Ae^{kt}$

$t = 0, y = 2 \Rightarrow A = 2$.

$y = 2 \cdot e^{kt}$ $(t = 2, y = 4)$

$4 = 2 \cdot e^{2k}$

$\ln 2 = 2k \Rightarrow k = \frac{\ln 2}{2}$.

$\Rightarrow \boxed{y = 2e^{\frac{\ln 2}{2} t}}$.

$t = 3 \Rightarrow y = 2 \cdot e^{\left(\frac{\ln 2}{2}\right)(3)}$

$\approx \boxed{5.66}$

#3. V: volume (ft³) t: time (second)

$\frac{dV}{dt} = kV$

$\int \frac{dV}{V} = \int k\,dt$

$\ln V = kt + C \Rightarrow V = e^{kt} \cdot e^{C} = A \cdot e^{kt}$

$V = A \cdot e^{kt}$ $\boxed{A = 36\pi}$ when $t=0$

$V = 36\pi e^{kt}$ $t=1$ $V = 90\pi$

$90\pi = 36\pi e^{k}$

$\ln \frac{90}{36} = k$

$\Rightarrow V = 36\pi e^{\ln(\frac{90}{36})t} = 36\pi e^{0.9163t}$

$V = 36\pi e^{\ln(\frac{90}{36})\cdot(3)} \approx \boxed{1767.1458\ ft^3}$

$\approx \boxed{1770\ ft^3}$

#4. a) $\frac{dS}{dt} = -0.1152S$

$\int \frac{dS}{S} = \int -0.1152\,dt$

$\ln S = -0.1152t + C$

$\boxed{S = A \cdot e^{-0.1152t}}$

$\boxed{\frac{S}{A} = e^{-0.1152(20)}}$

$\frac{S}{A} = 0.0998$

$\boxed{\frac{S}{A} = 9.99\%}$ of the initial amount.

b) Half Life. (How long it takes the substance becomes the half amount)

$\ln \frac{1}{2} = \ln e^{-0.1152t}$

$t = \frac{\ln(0.5)}{-0.1152} \approx \boxed{6.02\text{ yrs}}$