IB Math HL 1: Definite Integral using U-substitution Name: key.

Warm UP: Given $f(x) = 4x(2x^2+1)$, $=(8x^3+4x)$

a. Find the area A of the region between y=f(x) and x-axis on the interval [0, 2] using left hand rectangle and n= 20. Show proper sigma notation and use Graphing calculator to evaluate.

b. Find $\int 4x(2x^2+1)dx$ ⇐ Indefinite Integral.

Definite Integral Using U-Substitution:

Find the exact area of A using the Fundamental Theorem of calculus **using U-substitution.**

1. $\int_1^3 4x(2x^2+1)dx \Rightarrow \int_3^{19} u \cdot du$

$u = 2x^2+1$

$du = 4x\,dx$

$x=1 \Rightarrow u = 2\cdot1^2+1 = 3$

$x=3 \Rightarrow u = 2\cdot3^2+1 = 19$

$= \frac{1}{2}u^2 \Big|_{u=3}^{u=19}$

$= (\frac{1}{2})(19)^2 - \frac{1}{2}(3)^2$

$= \boxed{176}$

2. $\int_1^3 \frac{x}{\sqrt{3x^2+2}}dx = \int_5^{29} \frac{1}{6} u^{-\frac{1}{2}} \cdot du.$

$u = 3x^2+2$

$du = 6x \cdot dx$

$\frac{1}{6}du = x \cdot dx$

$x=1 \Rightarrow u=5$

$x=3 \Rightarrow u = 3\cdot3^2+2 = 29.$

$= \frac{1}{3}\sqrt{u}\Big|_{u=5}^{u=29}$

$= \frac{1}{3}\sqrt{29} - \frac{1}{3}\sqrt{5}$

$= \boxed{\frac{\sqrt{29}-\sqrt{5}}{3}}$

Practice)

$\int_0^1 2x(x^2+5)dx \Rightarrow \int_5^6 u\,du$

$u = x^2+5$

$du = 2x\,du.$

$x=0 \Rightarrow u=5$

$x=1 \Rightarrow u=6.$

$= \frac{1}{2}u^2\Big|_{u=5}^{u=6}$

$= \frac{1}{2}(6^2-5^2) = \boxed{\frac{11}{2}}$

36
-25

IB Math HL1 Exit Slip # 2 Name: ____________________

Find the indefinite integral (Anti-derivatives).

1. $\int(\sqrt[3]{x}+2e^{-4x})dx$ $= \int(x^{\frac{1}{3}}+2e^{-4x})dx$

$= \frac{3}{4}x^{4/3} - \frac{2}{4}e^{-4x} + c$

$= \boxed{\frac{3}{4}x\sqrt[3]{x} - \frac{1}{2}e^{-4x} + c}$

2. $\int\sec^4 xdx$ $= \int \sec^2 x \cdot \sec^2 x\, dx$

$= \int(1+\tan^2 x)\cdot \boxed{\sec^2 x\, dx} = \int(1+u^2)du$

$u = \tan x$. $du = \sec^2 x\, dx$. $= u + \frac{1}{3}u^3 + c$

$= \boxed{\tan x + \frac{1}{3}\tan^3 x + c}$

3. $\int\frac{x}{\sqrt{3x^2+2}}dx$

$u = 3x^2+2$

$du = 6x\,dx$

$\frac{1}{6} = x\,dx$

$\Rightarrow \int \frac{1}{6}u^{-\frac{1}{2}}du$

$= \boxed{\frac{1}{3}\sqrt{3x^2+2} + c}$

4. $\int\frac{\sin(2x+1)}{\cos^2(2x+1)}dx$

$u = \cos(2x+1)$

$du = -2\sin(2x+1)dx$

$-\frac{1}{2}du = \sin(2x+1)dx$

$\Rightarrow \int \frac{-\frac{1}{2}du}{u^2}$

$= \frac{1}{2}u^{-1} + c$

$= \boxed{\frac{1}{2\cos(2x+1)} + c}$

5. $\int x\sin(5+2x^2)dx$

$u = 2x^2+5$

$du = 4x\,dx$

$\frac{1}{4}du = x\,dx$

$\int\frac{1}{4}(\sin u)du = -\frac{1}{4}\cos u + c$

$= \boxed{-\frac{1}{4}\cos(2x^2+5) + c}$

6. $\int\frac{5+x^2}{1+x^2}dx$

$\int \frac{x^2+1+4}{x^2+1}dx$

$= \int(1 + \frac{4}{x^2+1})dx$

$= \boxed{x + 4\arctan x + c}$

7. Evaluate $\int_0^{\sqrt{3}}\frac{\arctan x}{1+x^2}dx$. Give your answer in exact.

$u = \arctan x$

$du = \frac{1}{1+x^2}dx$

$x = 0 \Rightarrow u = 0$

$x = \sqrt{3}$ $\arctan\sqrt{3} = \frac{\pi}{3}$

$\int_0^{\frac{\pi}{3}} u\,du$

$= \frac{1}{2}u^2 \Big|_{u=0}^{u=\frac{\pi}{3}}$

$= \boxed{\frac{1}{2}\left(\frac{\pi}{3}\right)^2}$