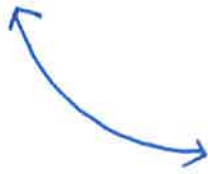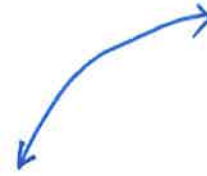IB Math HL1 Graph   *Warm-up*   Name_____ *key*

1. Sketch functions as indicated. Fill in the blanks with increasing, decreasing, positive, or negative.

a. Sketch a function $f(x)$ that is decreasing and concave up.

b. Sketch a function $g(x)$ that is increasing and concave down.

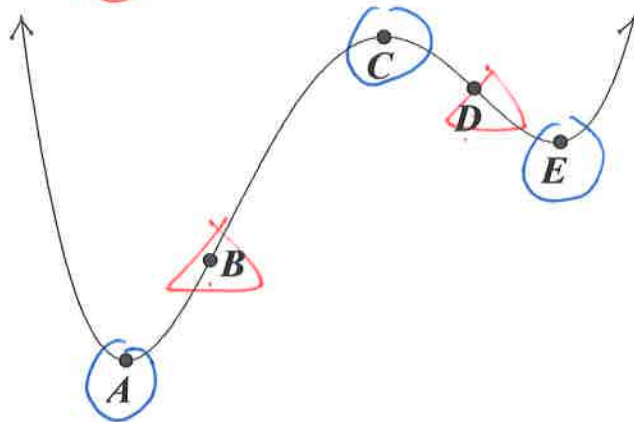$f'(x)$ is ___*less than* 0___ $f''(x)$ is ___*greater than* 0___
$\quad\quad\quad\quad$ (−) $\quad\quad\quad\quad\quad\quad$ (+)

$g'(x)$ is ___> 0___ $g''(x)$ is ___< 0___
$\quad\quad\quad\quad$ (+) $\quad\quad\quad\quad\quad\quad$ (−)

2. Consider the function $h(x)$.

Make a sign diagram for $h'(x)$.

$$\begin{array}{ccccc} & A & C & E & \\ \leftarrow & \circ & \circ & \circ & \rightarrow \\ (-) & & (+) & (-) & (+) \end{array}$$

Make a sign diagram for $h''(x)$.

$$\begin{array}{ccc} & B & D \\ \leftarrow & \circ & \circ & \rightarrow \\ (+) & & (-) & (+) \end{array}$$
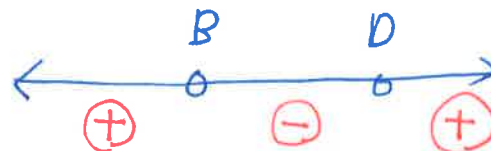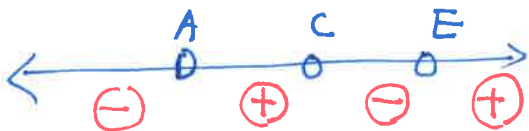
Ex) Sketch of graph $f(x) = x^4 - 4x^3 + 2$

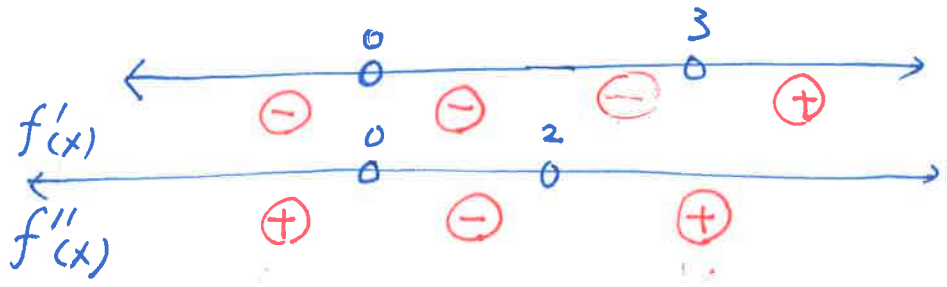a. Make the sign diagrams of the first derivative and second derivative.

$\frac{df}{dx} = 4x^3 - 12x^2 = 0 \implies 4x^2(x-3) = 0 \quad \boxed{x = 0} \quad x = 3$

Critical points

$\frac{d^2f}{dx^2} = 12x^2 - 24x \implies 12x(x-2) = 0 \quad x = 2 \quad \boxed{x = 0}$

Inflection point.

$f'(x)$ number line: 0, 3; signs: $\ominus$ (at 0), $\ominus$ (at 2), $\ominus$, $\oplus$

$f''(x)$ number line: 0, 2; signs: $\oplus$, $\ominus$, $\oplus$

a. Give the intervals where $f(x)$ is

Increasing	Decreasing	Concave Up	Concave Down
$(3, \infty)$	$(-\infty, 0) \cup (0, 3)$	$(-\infty, 0) \cup (2, \infty)$	$(0, 2)$

critical

b. Give coordinates where $f(x)$ has a

Local maximum	Local minimum	Stationary Inflection	Non-stationary inflection
None	$x = \boxed{3}$	$x = 0$ $y = 2$	$x = 2$ $y = -14$

$y = (3)^4 - 4(3)^3 + 2 = 3^3(3-4) + 2$

$= -27 + 2 = \boxed{-25}$

$(3, -25)$

c. Sketch a graph of $f(x)$

$(0,2)$
2
3
$I.P$
-25
$c.P$
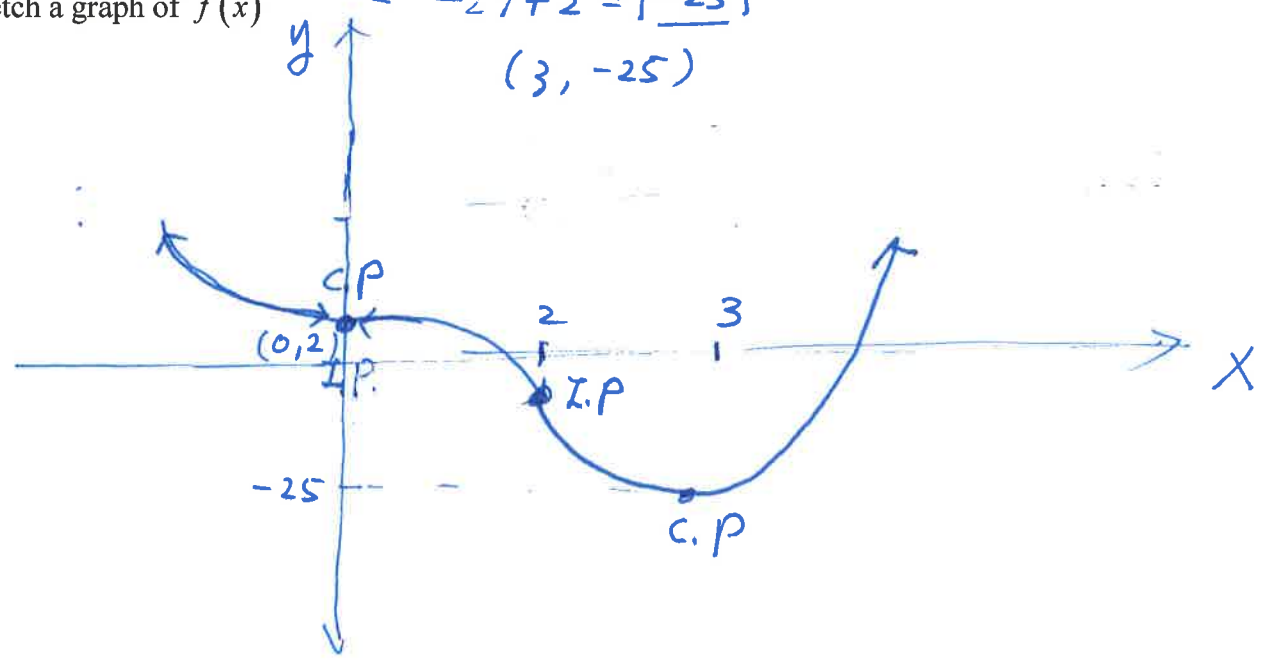
Practice)

1. Given the graph of $f(x)$, sketch the graphs of $f'(x)$ and $f''(x)$.

$f''(x) < 0$ $y = f(x)$ $f''(x) > 0$ $y = f'(x)$ $y = f''(x)$

$f'(x) < 0$ $f''(x) < 0$

-5 -3 0 3 $5\ x$ -5 $5\ x$ -5 $5\ x$

$f''(x) = 0$

$f'(x) > 0$
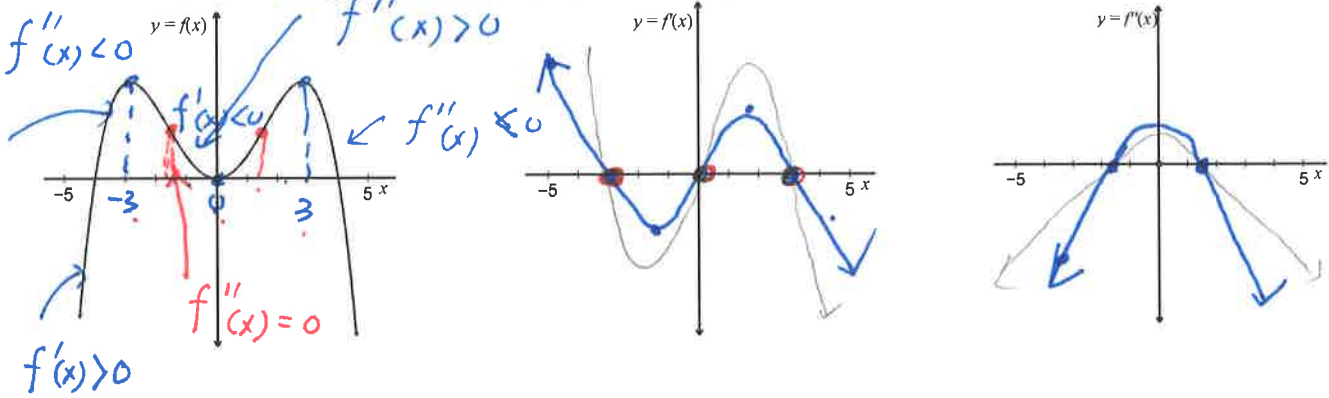
2. Use the graph of f' to sketch a graph of f and the graph of f''.

$y = f(x)$ $y = f'(x)$ $y = f''(x)$

$f'(x) > 0$ $f(x): Min$

-5 $5\ x$ -5 $5\ x$ -5 $5\ x$

$x = -2$ $x = 2$

$f(x): Max$ $f'(x) < 0$ $x = 0$

$f(x): Inflection point$.
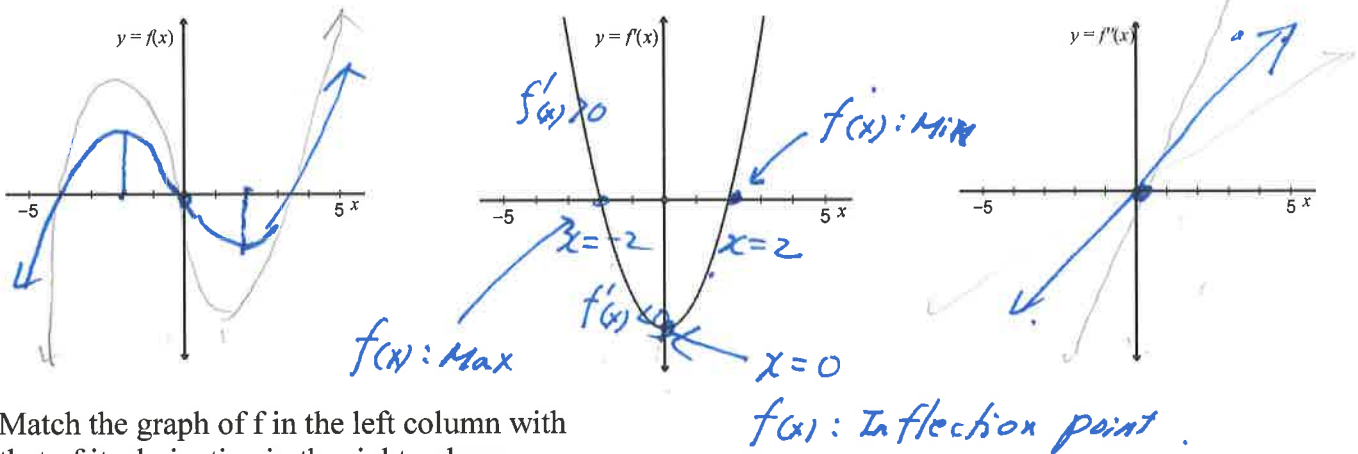
Match the graph of f in the left column with
that of its derivative in the right column.
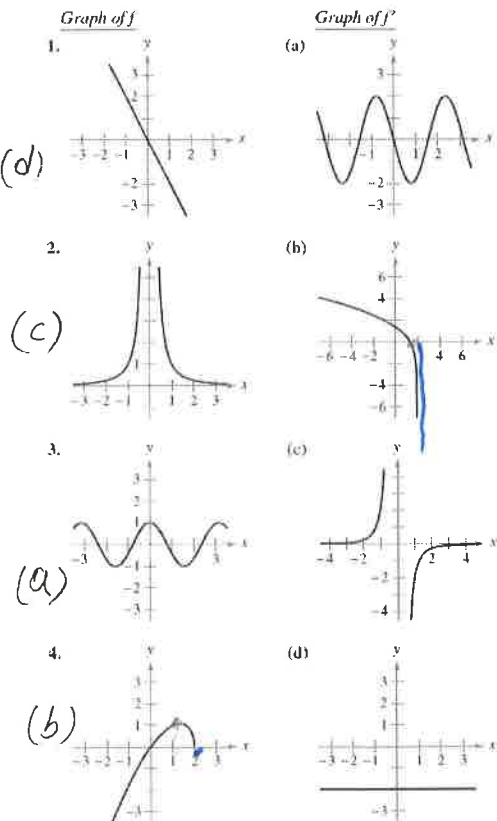
Use the graph of f' to
a. identify the interval(s) on which
Given the graph of f, sketch the graph of f'.
      f is increasing or decreasing
b. estimate the values of x at which
      f has a relative maximum
      or minimum

Graph of f   Graph of f'

(d) 1.   (a)

(c) 2.   (b)

5.   6 $f'(x)$

$f(x)$ f''

(a) 3.   (c)

7.

(b) 4.   (d)

$f'(x)$

$f'(-4)$ is undefined.