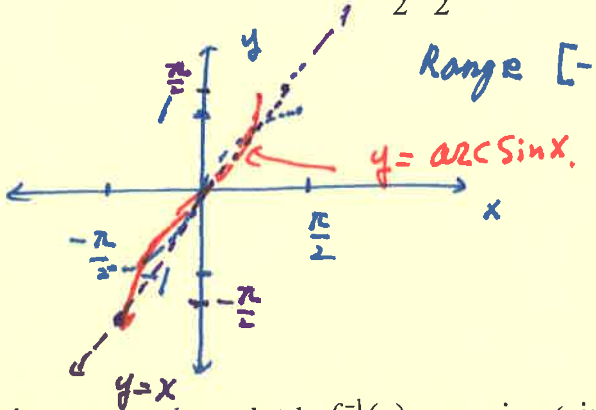
IB Math HL1 (Derivatives of Inverse Trig Functions) **Name:** Key **Period:** _______

Inverse sine function:

1. Sketch $f(x)=\sin x$, where $x\in[-\frac{\pi}{2},\frac{\pi}{2}]$



Range [-1, 1]

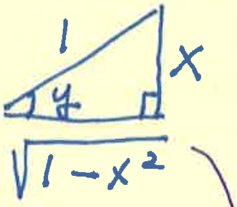
2. At the same axes above, sketch $f^{-1}(x)=arc\sin x\,(\sin^{-1}x)$
3. What is domain and range of $f^{-1}(x)$?

Domain: $[-1, 1]$ Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

4. Show $\frac{d}{dx}(\sin^{-1}x)=\frac{1}{\sqrt{1-x^2}}$ where $x\in[-1,1]$

$\boxed{y=\sin^{-1}x} = \arcsin x$

$\frac{x}{1}=\sin y$



$\frac{d}{dx}x=\frac{d}{dx}\sin y \Rightarrow \frac{dy}{dx}=?$

$1=\cos y\cdot\boxed{\frac{dy}{dx}}$

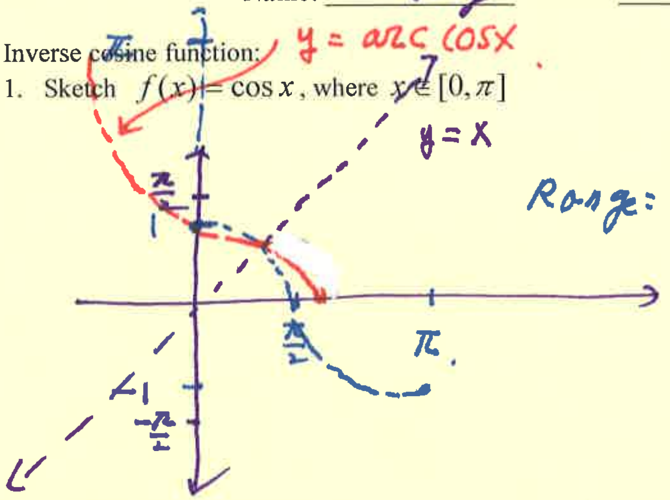
$\frac{dy}{dx}=\frac{1}{\cos y}.$ $\left(\cos y=\frac{\sqrt{1-x^2}}{1}\right)$

$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$

$\therefore \boxed{\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}}}$

Inverse cosine function:

1. Sketch $f(x)=\cos x$, where $x\in[0,\pi]$



Range: [-1, 1]

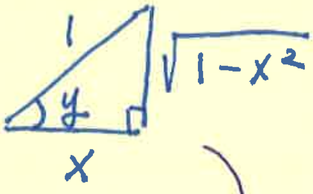
2. At the same axes above, sketch $f^{-1}(x)=\arccos x\,(\cos^{-1}x)$
3. What is domain and range of $f^{-1}(x)$?

Domain: $[-1, 1]$ Range: $[0,\pi]$

4. . $\frac{d}{dx}(\cos^{-1}x)=-\frac{1}{\sqrt{1-x^2}}$ where $x\in[-1,1]$

$y=\cos^{-1}x$

$\frac{x}{1}=\cos y$



$\frac{d}{dx}x=\frac{d}{dx}\cos y \Rightarrow \frac{dy}{dx}=?$

$1=-\sin y\cdot\boxed{\frac{dy}{dx}}.$

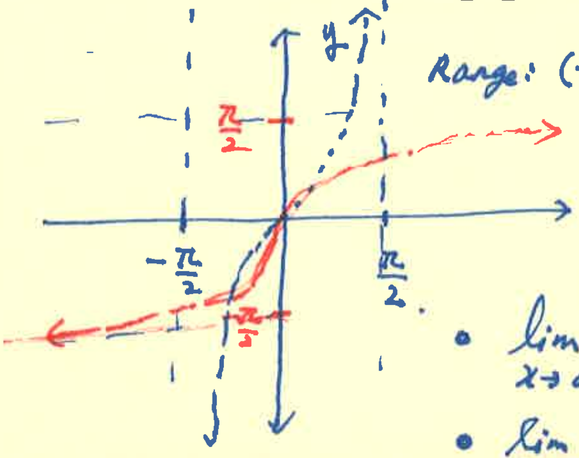
$\frac{dy}{dx}=\frac{1}{-\sin y}.$ $\left(\sin y=\sqrt{1-x^2}\right)$

$\boxed{\frac{dy}{dx}=-\frac{1}{\sqrt{1-x^2}}}$

$\therefore \boxed{\frac{d}{dx}\cos^{-1}x=\frac{-1}{\sqrt{1-x^2}}}$

Inverse tangent function:

1. Sketch $f(x) = \tan x$, where $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$



Range: $(-\infty, \infty)$

- $\lim_{x\to\infty} \tan^{-1}x = \frac{\pi}{2}$.
- $\lim_{x\to-\infty} \tan^{-1}x = -\frac{\pi}{2}$

2. At the same axes above, sketch $f^{-1}(x) = \arctan x\ (\tan^{-1} x)$
3. What is domain and range of $f^{-1}(x)$?

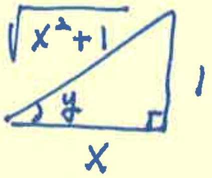
Domain: $(-\infty, \infty)$ Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$

Inverse Cotangent function.

Show $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

$y = \cot^{-1} x$

$x = \cot y$



$\frac{d}{dx} x = \frac{d}{dx} \cot y$

$1 = -\csc^2 y \cdot \frac{dy}{dx}$

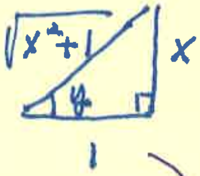
$\frac{dy}{dx} = \frac{1}{-\csc^2 y}$ $(\csc y = \sqrt{x^2+1}$

$$\boxed{\frac{dy}{dx} = -\frac{1}{x^2+1}}$$

4.. Show $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ where $x \in R$

$y = \tan^{-1} x.$ $\Rightarrow$ $\frac{dy}{dx} = ?$

$x = \tan y.$



$\frac{d}{dx} x = \frac{d}{dx} \tan y$

$1 = \sec^2 y \cdot \frac{dy}{dx}.$

$\frac{dy}{dx} = \frac{1}{\sec^2 y}$ $(\sec y = \sqrt{x^2+1})$

$$\boxed{\frac{dy}{dx} = \frac{1}{x^2+1}}$$

$$\therefore \boxed{\frac{d}{dx} \tan^{-1}x = \frac{1}{1+x^2}}$$

Optional: Prove the following derivatives:

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$

$\Rightarrow$ See attached.

Practice)

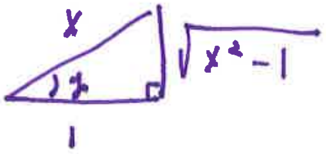
1. $y = \cos^{-1}(x^2)$
2. $y = \sin^{-1}(\sqrt{2x})$
3. $y = \sec^{-1}(2x+1)$
4. $y = \csc^{-1}(x^2+1)$
5. $y = (\cot^{-1}\sqrt{x-1})^5$
6. $y = \tan^{-1}\sqrt{x^2-1} + \csc^{-1} x$
7. $y = 8^x$
8. $y = 3^{\cot x}$
9. $y = (\ln x)^2$
10. $y = \log_4 x^2$
11. $y = \frac{1}{\log_2 x}$
12. $y = \log_3(1 + x\ln 3)$

Anwers attached.

① Show $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$

$y = \sec^{-1}x$

$x = \sec y \quad \Rightarrow$



$\frac{dx}{dx} = \frac{d}{dx}\sec y$

$1 = \sec y \tan y \cdot \frac{dy}{dx}$.

$\frac{dy}{dx} = \frac{1}{\sec y \cdot \tan y}$ $\left(\sec y = x \ , \ \tan y = \sqrt{x^2-1} \right)$

$\boxed{\frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}}}$

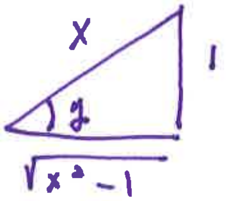
x is hypotenus .

so x must be positive ⇒ $|x|$

② Show $\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$

$y = \csc^{-1}x$

$x = \csc y \quad \Rightarrow$



$\frac{dx}{dx} = \frac{d}{dx}\csc y$

$1 = -\csc y \cdot \cot y \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-1}{\csc y \cdot \cot y}$ $\left((\csc y = x \quad \cot y = \sqrt{x^2-1} \right)$

$\boxed{\frac{dy}{dx} = \frac{-1}{|x|\sqrt{\quad}}}$

practice) Inverse trig derivative WS. key

#1. $y' = \dfrac{-2x}{\sqrt{1-x^4}}$

#2. $y' = \dfrac{1}{\sqrt{1-2x}}\left(\cancel{\tfrac{1}{2}}\cdot\cancel{(2)}(2x)^{-\frac{1}{2}}\right)$

$\boxed{y' = \dfrac{1}{\sqrt{2x}\sqrt{1-2x}}}$

#3. $\boxed{y' = \dfrac{2}{|2x+1|\sqrt{(2x+1)^2-1}}}$

#4. $\boxed{y' = \dfrac{-2x}{|x^2+1|\sqrt{(x^2+1)^2-1}}}$

#5. $y' = -5\left[\cot^{-1}\sqrt{x-1}\right]^4\cdot\left(\frac{1}{x}\right)\left(\frac{1}{2\sqrt{x-1}}\right)$

$\cancel{+\dfrac{-1}{|x|\sqrt{x^2-1}}}$ $\quad$ $\cancel{-\dfrac{1}{|x|\sqrt{x^2-1}}}$

#6. $y' = \dfrac{\left(\cancel{\frac{1}{2}}\right)(x^2-1)^{-\frac{1}{2}}(\cancel{2}x)}{\cancel{1}+(x^2\cancel{-1})} = \dfrac{x}{x^2\sqrt{x^2-1}} = \boxed{\dfrac{1}{x\sqrt{x^2-1}}} - \boxed{\dfrac{1}{|x|\sqrt{x^2-1}}} = \boxed{0}$

#7. $\boxed{y' = \ln 8\cdot 8^x}$

#8. $\boxed{y' = (\ln 3)\cdot 3^{\cot x}(-\csc x)^2}$

#9. $\boxed{y' = 2(\ln x)\frac{1}{x}}$

#10 $y' = \dfrac{2x}{(\ln 4)(x^2)} = \boxed{\dfrac{2}{(\ln 4)x}}$

#11. $y \overset{*}{=} (\log_2 x)^{-1}$

$y' = -1(\log_2 x)^{-2}\left(\dfrac{1}{(\ln 2)x}\right)$

$\boxed{y' = \dfrac{-1}{(\log_2 x)^2(\ln 2)x}}$

#12.

$y' = \left(\dfrac{1}{\cancel{\ln 3}}\right)\left(\dfrac{1}{1+x\ln 3}\right)(\cancel{\ln 3})$

$y' = \boxed{\dfrac{1}{1+x\ln 3}}$