

18G: Derivatives of Log Functions

The derivative of $y = \ln x$.

$$y = \ln x = \log_e x$$

$$\Rightarrow \frac{d}{dx} x = \frac{d}{dx} e^y \Rightarrow x = e^y$$

$$\Rightarrow 1 = e^y \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

The derivative of $y = \log_b x$

$$y = \log_b x \Rightarrow x = b^y$$

$$\Rightarrow \frac{d}{dx} x = \frac{d}{dx} b^y$$

$$\Rightarrow 1 = b^y \cdot \ln b \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(\ln b) \cdot b^y} = \frac{1}{(\ln b) \cdot x}$$

$$\frac{\ln x}{\ln b} = \frac{\log x}{\log b}$$

$$y = \left(\frac{1}{\ln b}\right) \cdot \ln x$$

$$\frac{dy}{dx} = \left(\frac{1}{\ln b}\right) \cdot \frac{1}{x}$$

$\frac{d}{dx}(\log_b x) = \frac{1}{(\ln b) \cdot x}$	$\frac{d}{dx}(\ln x) = \frac{1}{x}$
$\frac{d}{dx}(\log_b u) = \frac{d}{du}(\log_b u) \cdot \frac{du}{dx} = \frac{1}{(\ln b) \cdot u} \cdot \frac{du}{dx}$	$\frac{d}{dx}(\ln u) = \frac{d}{du}(\ln u) \cdot \frac{du}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$

Examples: Find $\frac{dy}{dx} = \frac{1}{(\ln b) \cdot u} \cdot \frac{du}{dx}$

1) $y = \ln(5x^3 - 4x)$

$$\frac{dy}{dx} = \left(\frac{1}{5x^3 - 4x}\right) (15x^2 - 4) = \frac{15x^2 - 4}{5x^3 - 4x}$$

2) $y = (\log_6 2x - 7^{\sqrt{x}})^5$

7^u ($u = \sqrt{x}$)

$$\frac{dy}{dx} = 5(\log_6 2x - 7^{\sqrt{x}})^4 \left[\frac{1 \cdot 2}{(\ln 6)(2x)} - 7^{\sqrt{x}} \cdot (\ln 7) \cdot \frac{1}{2} x^{-\frac{1}{2}} \right]$$

$$= 5(\log_6 2x - 7^{\sqrt{x}})^4 \left[\frac{1}{(\ln 6) \cdot x} - \frac{7^{\sqrt{x}} \cdot (\ln 7)}{2\sqrt{x}} \right]$$