Notes: Derivatives of $(\ln x)$, (e^x), (b^x), and $(\log_b x)$

1) $\frac{d}{dx}(\ln x) = \boxed{\frac{1}{x}}$

$\lim_{h\to 0}\frac{\ln(x+h)-\ln x}{h} = \lim_{h\to 0}\frac{\ln\left(\frac{x+h}{x}\right)}{h}$	Use of the first principle of derivative $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$
$\lim_{u\to\infty}\frac{u}{x}\left[\ln\left(1+\frac{h}{x}\right)\right]$ $\left(u=\frac{x}{h}\Rightarrow\frac{h}{x}=\frac{1}{u};\ h=\frac{x}{u}\Rightarrow\frac{1}{h}=\frac{u}{x}\right)$ $=\lim_{u\to\infty}\frac{u}{x}\left[\ln\left(1+\frac{1}{u}\right)\right]$ $=\frac{1}{x}\lim_{u\to\infty}u\left(\ln\left(1+\frac{1}{u}\right)\right)$: x is independent of u.	Let $u=\frac{x}{h}\Rightarrow h=\frac{x}{u}$ and As $h\to 0\Rightarrow u\to\infty$ Rewrite the derivative in terms of u variable.
$=\frac{1}{x}\lim_{u\to\infty}\ln\left(1+\frac{1}{u}\right)^u=\frac{1}{x}\ln\left[\lim_{u\to\infty}\left(1+\frac{1}{u}\right)^u\right]$	Simplify
$=\frac{1}{x}\cdot\ln e=\boxed{\frac{1}{x}}$	Substitute $\left(\lim_{u\to\infty}\left(1+\frac{1}{u}\right)^u\right)=e$

2) $\frac{d}{dx}(e^x) = \boxed{e^x}$

$v=e^x\Rightarrow\ln v=\ln e^x=x$	Substitute $v=e^x$ and isolate x.
$\frac{d}{dx}x=\frac{d}{dx}\ln v\Rightarrow 1=\frac{1}{v}\frac{dv}{dx}$ (Chain Rule and #1)	Differentiate both sides
$\frac{dv}{dx}=v=\boxed{e^x}$	Solve for $\frac{dv}{dx}$

3) $\frac{d}{dx}(b^x) = \boxed{\ln b\cdot b^x}$

$b^x=e^{\ln b^x}=e^{x\ln b}$	Express $b^x=e^{\ln(\quad)^x}$.
$\frac{d}{dx}(b^x)=\frac{d}{dx}e^{x\ln b}=\ln b\cdot e^{x\ln b}$	Differentiate both sides
$\frac{d}{dx}(b^x)=\ln b\cdot e^{\ln b^x}=\boxed{\ln b\cdot b^x}$	Simplify

3) $\frac{d}{dx}(\log_b x) = \boxed{\frac{1}{\ln b \cdot x}}$

$\log_b x = \frac{\ln x}{\ln b} = \frac{1}{\ln b} \cdot \ln x$	Express $(\log_b x) = \frac{\ln(\quad)}{\ln(\quad)}$.
$\frac{d}{dx}(\log_b x) = \frac{d}{dx}\left(\frac{1}{\ln b} \cdot \ln x\right) = \frac{1}{\ln b} \cdot \frac{1}{x}$ (from #1)	Differentiate both sides
$\frac{d}{dx}(\log_b x) = \boxed{\frac{1}{(\ln b) \cdot x}}$	Simplify

Examples : Find $\frac{df}{dx}$.

1. $f(x) = e^{2x^2-3}$

$\boxed{\frac{df}{dx} = (4x)\left(e^{2x^2-3}\right)}$

2. $f(x) = x^2 e^x$

$\boxed{\frac{df}{dx} = 2xe^x + x^2e^x}$

3. $f(x) = \frac{e^x}{x^2}$

$\frac{df}{dx} = \frac{e^x \cdot x^2 - 2xe^x}{x^4}$

$= \boxed{\frac{e^x \cdot x - 2e^x}{x^3}}$

4. $f(x) = (x^2 - 3x)^2 e^{-5x}$

$\frac{df}{dx} = \boxed{2(x^2-3x)(2x-3)e^{-5x} - 5(x^2-3x)^2e^{-5x}}$

Practice) Find $\frac{df}{dx}$ Answers are attached.

5. $f(x) = e^{\sqrt{2x-3}}$

6. $f(x) = \frac{\sqrt{x}(x^3 - 2x)}{e^x}$

7. $f(x) = 3^{2x-5}$

8. $f(x) = e^{x^2} 5^x$

#5.

$$\frac{df}{dx} = (2)\left(\tfrac{1}{2}\right)(2x-3)^{-\frac{1}{2}} \cdot e^{\sqrt{2x-3}}$$

$$= \boxed{\frac{e^{\sqrt{2x-3}}}{\sqrt{2x-3}}}$$

#6. $$f(x) = \frac{x^{\frac{7}{2}} - 2x^{\frac{3}{2}}}{e^x}$$

$$\frac{df}{dx} = \frac{\left(\frac{7}{2}x^{\frac{5}{2}} - 3x^{\frac{1}{2}}\right)\cdot \cancel{e^x} - \cancel{e^x}\left(x^{\frac{7}{2}} - 2x^{\frac{3}{2}}\right)}{(e^x)^{\cancel{2}}}$$

$$= \boxed{\frac{\frac{7}{2}x^{\frac{5}{2}} - 3x^{\frac{1}{2}} - x^{\frac{7}{2}} + 2x^{\frac{3}{2}}}{e^x}}$$

#7. $$\boxed{\frac{df}{dx} = (2)(\ln 3)\, 3^{2x-5}}$$

#8. $$\boxed{\frac{df}{dx} = 2x \cdot e^x \cdot 5^x + (\ln 5)(e^{x^2})5^x}$$