

IB Math 2: Even More Practice for Differential Equation

Name: Key Period: \_\_\_\_\_

1. Find a general solution to the differential equation;  $(9+9x) \frac{dy}{dx} = 0.75$ .

$$dy = .75(9+9x)^{-1} dx$$

$$y = \frac{1}{12} \ln(9+9x) + C$$

2. Solve  $\frac{dy}{dx} = 10xy^2$  given that  $y(1) = 1$ .

$$y^{-2} dy = 10x dx$$

$$-\frac{1}{y} = 5x^2 + C$$

$$-1 = 5 + C$$

$$C = -6$$

3. Given that  $t^3 \cdot \frac{d\theta}{dt} = k - t$  and  $\theta(2) = 0 = \theta(4)$ , find  $k$ .

$$d\theta = (k-t)t^{-3} dt$$

$$\theta = kt^{-3} - t^{-2} + C$$

$$\theta = -\frac{1}{2}kt^{-2} + t^{-1} + C$$

$$0 = -\frac{1}{2}k\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right) + C$$

$$0 = -\frac{1}{8}k + \frac{1}{2} + C$$

$$\frac{1}{8}k = \frac{1}{2} + C \quad k = 4 + 8C$$

$$0 = -\frac{1}{2}k\left(\frac{1}{16}\right) + \frac{1}{4} + C$$

$$0 = -\frac{1}{32}k + \frac{1}{4} + C$$

$$\frac{1}{32}k = \frac{1}{4} + C$$

$$k = 8 + 32C$$

$$4 + 8C = 8 + 32C$$

$$-4 = 24C$$

$$-\frac{1}{6} = C$$

$$k = 4 - \frac{8}{6}$$

$$k = \frac{16}{6}$$

$$k = \frac{8}{3}$$

$$\theta = -\frac{4}{3}t^{-2} + t^{-1}$$

4. Water flows from a hole in the base of a cylindrical can at a rate given by  $\frac{dh}{dt} = -k\sqrt{h}$  where  $k > 0$  and  $h$  is the depth of water in the vessel at time  $t$ . If the level of water in the can falls from 100 cm to 25 cm in two minutes, how much longer will it take for the can to empty?

$$h^{-\frac{1}{2}} dh = -k dt$$

$$2h^{\frac{1}{2}} = -kt + C$$

$$2(100)^{\frac{1}{2}} = -k(0) + C$$

$$C = 20$$

$$2(25)^{\frac{1}{2}} = -k(2) + 20$$

$$10 = -2k + 20$$

$$-10 = -2k$$

$$k = 5$$

$$2\sqrt{h} = -5t + 20$$

$$0 = -5t + 20$$

$$5t = 20$$

$$t = 4 \text{ min}$$

or 2 min longer

5. Solve the differential equation  $\frac{d^2x}{dt^2} = 2\sin 2t - 4\cos 2t$ , if  $\frac{dx}{dt}(0) = 2$  and  $x(0) = 1$

$$d^2x = 2\sin 2t - 4\cos 2t dt^2$$

$$dx = -\cos 2t - 2\sin 2t + C dt$$

$$2 = -(\cos(0)) - 2\sin(0) + C$$

$$2 = -1 + C$$

$$C = 3$$

$$dx = -\cos 2t - 2\sin 2t + 3 dt$$

$$x = -\frac{1}{2}\sin 2t + \cos 2t + 3t + C$$

$$1 = -\frac{1}{2}\sin(0) + \cos(0) + 0 + C$$

$$1 = 1 + C$$

$$C = 0$$

$$x = -\frac{1}{2}\sin 2t + \cos 2t + 3t$$