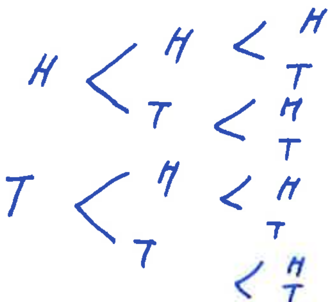
Discrete Random Variable:

Is one in which we can produce a countable number of outcomes. (A random variable represents in number form the possible outcomes which could occur for some random experiment.)

$P(X=x)$ or P_x; (the probability that the random variable X=x is P)

Ex) Consider the experiment of tossing a coin three times in succession. If the random variable x denotes the number of heads observed, list the values that x can have and find the corresponding probability values.



$\Rightarrow P(HHH) = \frac{1}{8}$

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$\rightarrow P(HTT) = \frac{1}{8}$

$\rightarrow P(THH) = \frac{1}{8}$

$P(THT) = \frac{1}{8}$

$P(TTH) = \frac{1}{8}$

$P(TTT) = \frac{1}{8}$

$P(X=0) = \frac{1}{8}$

$P(X=1) = \frac{3}{8}$

$P(X=2) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$

Probability Distribution

- **Tabular Form:**

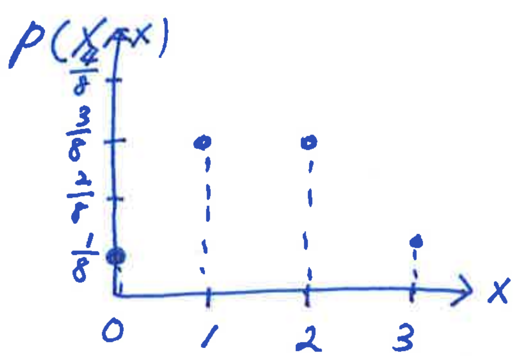
x	0	1	2	3
P(X=x)	1/8	3/8	3/8	1/8

$\Leftarrow$ ☆

- **Function Form:** $P(X=x)=\binom{3}{x}(\frac{1}{2})^3$ **where x= 0, 1, 2, 3** $\Leftarrow$ ☆

$= {}_3C_x \left(\frac{1}{2}\right)^3$

- **Graphical Representation:**



(X: #s of Head)

Prosperities of the Probability Function:

1) $0 \le P(X=x_i) \le 1$

2) $\sum_{i=0}^{i=n} P(X=x_i) = 1$

Ex) The probability distribution of the random variable x is represented by the function P(X=x) = k/x, where x = 1, 2, 3, 4, 5, 6.

a) Find the value of k.

$\frac{k}{1} + \frac{k}{2} + \frac{k}{3} + \frac{k}{4} + \frac{k}{5} + \frac{k}{6} = 1 \Rightarrow \frac{60k + 30k + 20k + 15k + 12k + 10k}{60} = 1$

$k = \frac{60}{147} = \boxed{\frac{20}{49}}$

b) Find $P(3 \le x \le 5)$

$P(X=3) + P(X=4) + P(X=5) = \frac{k}{3} + \frac{k}{4} + \frac{k}{5}$

$= \frac{47}{147}$

Ex) A discrete random variable x has a probability distribution defined by the $P(X = x) = \binom{4}{x}(\frac{2}{5})^x(\frac{3}{5})^{4-x}$ where x =0, 1, 2, 3, and 4.

$P(X=0) = {}_4C_0 \left(\frac{2}{5}\right)^0 \left(\frac{3}{5}\right)^4$

$= \left(\frac{3}{5}\right)^4 = \frac{81}{625}$

a) Display this distribution using a table form.

x	0	1	2	3	4
$P(X=x)$	$\frac{81}{625}$	$\frac{216}{625}$	$\frac{216}{625}$	$\frac{96}{625}$	$\frac{16}{625}$

b) Find $P(1 \le x \le 3)$.

$P(X=1) + P(X=2) + P(X=3) = 1 - P(X=0) - P(X=4)$

$= 1 - \frac{81}{625} - \frac{16}{625} = \frac{528}{625}$

Ex) A bag contains 5 white cube and 4 red cubes. Two cubes are selected in such a way that the first cube drawn is not replaced before the next cube is drawn. Find the probability distribution of x, where x denotes the number of white cubes selected from the bag.

X: Number of white cubes.

check: $\frac{1}{6} + \frac{5}{9} + \frac{5}{18} = 1.$

x	0	1	2
$P(X=x)$	$\frac{1}{6}$	$\frac{5}{9}$	$\frac{5}{18}$

- $P(X=0) = \left(\frac{4}{9}\right)\left(\frac{3}{8}\right) = \frac{1}{6}$
- $P(X=1) = \left(\frac{5}{9}\right)\left(\frac{4}{8}\right) + \left(\frac{4}{9}\right)\left(\frac{5}{8}\right) = \frac{5}{9}$
- $P(X=2) = \left(\frac{5}{9}\right)\left(\frac{4}{8}\right) = \frac{5}{18}$

Exit Slip: Name: key

1. Two friends, Andra and Diana, independently applied for different jobs. The chances that Andra is successful is 0.8 and the chances that Diana is successful is 0.75.

a) If X is denotes that number of successful applications between the two friends, find the probability distribution of x.

X: Number of Successful application.

X	0	1	2
$P(X=x)$			

$P(X=0) = (0.2)(0.25) = 0.05$

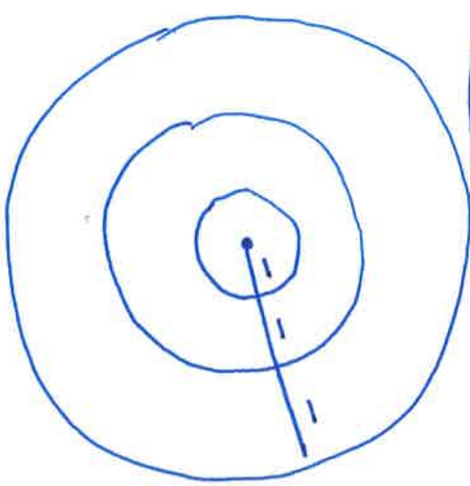
$P(X=1) = (0.8)(0.25) + (0.2)(0.75) = 0.2 + 0.15 = 0.35$

$P(X=2)\ (0.8)(0.75) = 0.6$

b) Find the probability that if one is successful, it is Andra.

$$P(X = \text{Andra's Success}) = \frac{0.2}{0.35} = \boxed{\frac{4}{7}}$$

2. **A dart board** consisting of concentric circle of radius 1, 2, and 3 units is placed against a wall. Upon throwing a dart, which lands at some random location on the board, a player will receive \$8.00 if the smaller circle is hit, \$6.00 if the middle annular region is hit, and \$4.00 if the outer annular region is hit. However, should the player miss the dart board altogether, they would lose \$7.00. The probability that the player missed the dart board is 0.5. In a long run, would it be a fair game for a player?



Expected Value:

$$\left[0.5\left[(8)\left(\frac{1^2\pi}{3^2\pi}\right) + (6)\left(\frac{(2^2-1^2)\pi}{3^2\pi}\right) + (4)\left(\frac{(3^2-2^2)\pi}{3^2\pi}\right)\right] - (0.5)(7)\right]$$

$$= \$\boxed{-0.5}$$

The player might loss \$ 0.50 (in a long run).