$\boxed{\pi \int_{x_1}^{x_2} f(x)^2\,dx}$ where $r = f(x)$.

IB Math HL1 : Examples of Disk Methods Name:_______________

Warm Up

1. The region bounded by , $y = x^2 + 1$, $x = 2$, and y axis is revolved about x-axis.

a) Draw a sketch of the solid and a representative slice of the disk.

b) Find the volume by the disk method.

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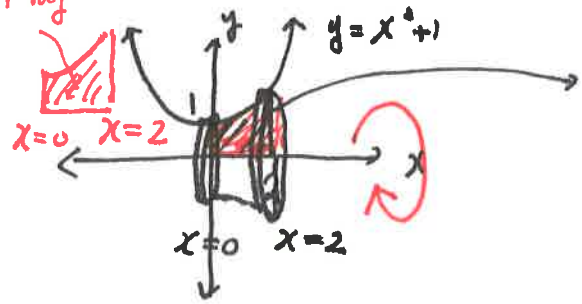
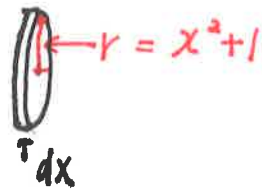




$$\Rightarrow V = \pi \int_0^2 (x^2+1)^2\,dx$$

$$= \pi \int_0^2 (x^4 + 2x^2 + 1)\,dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_{x=0}^{x=2}$$

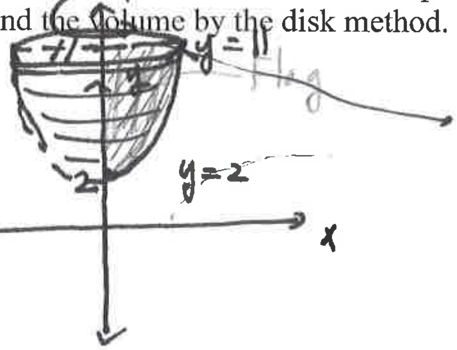
$$= \pi \left[\frac{32}{5} + \frac{16}{3} + 2 \right] = \boxed{\frac{206}{15}\pi}$$

Revolution about Y-axis. $\pi \int_{y_1}^{y_2} (f(y))^2\,dy$

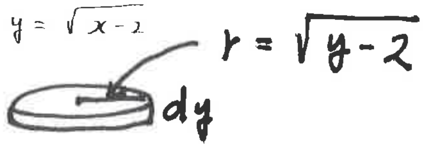
Example 1) The region, A, bounded by $x = \sqrt{y-2}$, $y = 11$, and y axis is revolved about y-axis.

a) Draw a sketch of the solid and a representative slice of the disk.

b) Find the volume by the disk method.



$y = \sqrt{x-2}$

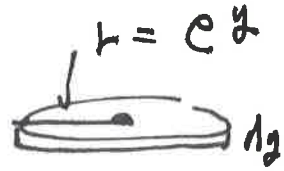


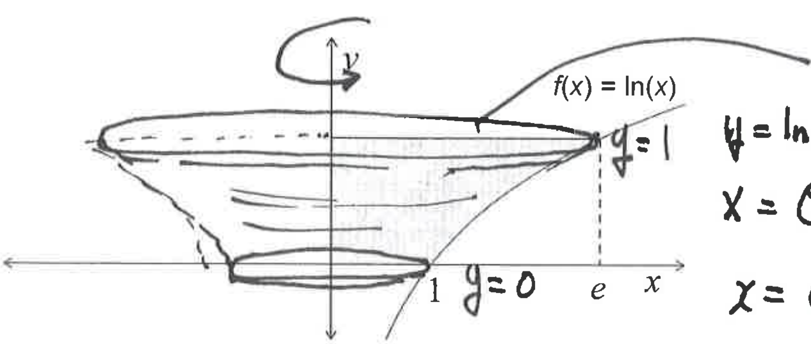
$$V = \pi \int_2^{11} (\sqrt{y-2})^2\,dy = \pi \int_2^{11} (y-2)\,dy$$

$$= \pi \left[\frac{1}{2}y^2 - 2y \right]_{y=2}^{y=11} = \pi\left(\left[\frac{11^2}{2} - 22 \right] - \left(\frac{2^2}{2} - 4 \right) \right)$$

$$= \boxed{40.5\pi} = \boxed{\frac{81}{2}\pi}$$

Example 2) The shaded region is revolved about the y-axis.
Find the volume of revolution.





$y = \ln x$

$x = e^y$

$x = e \Rightarrow y = \ln e = 1$

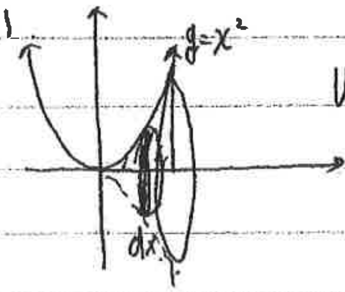
$$\boxed{\frac{\pi(e^2-1)}{2}}$$

$$\|$$

$$V = \pi \int_0^1 (e^y)^2\,dy = \pi \int e^{2y}\,dy = \pi \left[\frac{e^{2y}}{2} \right]_{y=0}^{y=1} = \pi \left[\frac{e^2}{2} - \frac{1}{2} \right]$$

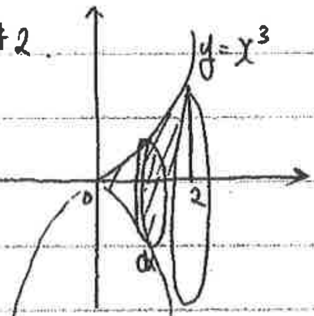
Volumes (The Disk Method)

#1



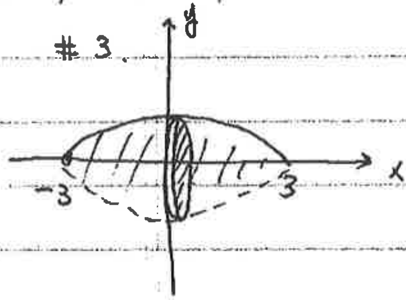
$$V=\int_0^2 \pi [x^2]^2 dx = \left[\frac{1}{5}x^5\right]_0^2 = \boxed{\frac{32}{5}\pi}$$

#2



$$V=\int_0^2 \pi[x^3]^2 dx = \int_0^2 x^6 dx = \left[\frac{1}{7}x^7\right]_0^2 = \frac{128}{7}\pi$$

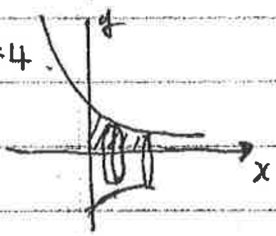
#3



$$V=2\int_0^3 \pi(\sqrt{9-x^2})^2 dx = 2\int(9-x^2)dx = 2\pi\left[9x-\frac{x^3}{3}\right]_0^3 = 36\pi$$

$$= 2\pi\cancel{\left[18-\frac{8}{3}\right]} = \cancel{\frac{92}{3}}\pi$$

#4



$$V=\int_0^1 \pi(e^{-x})^2 dx \quad \cancel{= \pi\int_0^1 e^{-x^2}dx}$$

$$\cancel{= \pi\int_0^1 \left[1+(-x^2)+\frac{(-x^2)^2}{2!}+\frac{(-x^2)^3}{3!}+\frac{(-x^2)^4}{4!}\right]dx}$$

$$\cancel{= \pi\left[x+\left(\frac{-x^3}{3}\right)+\frac{x^5}{10}-\frac{x^7}{42}+\frac{x^9}{216}\right]_0^1}$$

$\approx .7468\pi$

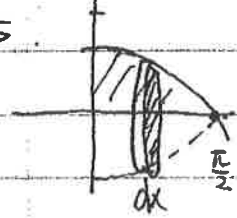
$(.74755\pi) \approx \boxed{\cancel{2.3485}}$

$$\Rightarrow V=\int_0^1 \pi(e^{-2x})dx$$

$$= \pi\left(-\frac{1}{2}e^{-2x}\right]_0^1 = \pi\left[\frac{-1}{2e^2}+\frac{1}{2}\right] = \boxed{\left(\frac{1}{2}-\frac{1}{2e^2}\right)\pi}$$

$$\approx \boxed{1.358}$$

#5



$$V=\int_0^{\pi/2} \pi(\sqrt{\cos x})^2 dx = \int_0^{\pi/2} \cos x\, dx$$

$$= \pi\left[\sin x\right]_0^{\frac{\pi}{2}} = \boxed{\pi}$$