IB Math 3: Divergence Test **Name:** key **Period:** _______

Warm-UP

Conditional, Converse, Inverse & Contrapositive Statements

A conditional statement is a statement that can be expressed in "if-then" form. Every conditional statement has three other conditionals associated with it. To get the **converse**, you switch the "if" and "then" parts. To get the **inverse**, you negate both parts. To get the **contrapositive**, you reverse and negate the two parts. These new conditionals may be true or false.

For each statement, write the missing statements. Then, determine if each new statement is true or false

Conditional	If you are in Seattle, then you are in the state of Washington.	True or False
Converse	If you are in the state of WA, then you are in Seattle	False
Inverse	If you are not in Seattle, then you are not in the state of WA	False
Contrapositive	If you are not in the state of WA, then you are not in Seattle	True

The following theorem states that if a series converges, the limit of the nth term converges to '0'.

Limit of the nth term of a convergence series theorem:

If $\sum_{k=1}^{\infty} a_k$ **converges, then** $\lim_{n\to\infty} a_n = 0$.

Proof:

Notes: $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n$

1) Suppose $\lim_{n\to\infty} S_n = L \Rightarrow$ This also implies $\lim_{n\to\infty} S_{n-1} = L$.

2) $S_n - S_{n-1} = [(a_1 + a_2 \cdots a_n) - (a_1 + a_2 \cdots a_{n-1})] = a_n$

3) $\lim_{n\to\infty} S_n - \lim_{n\to\infty} S_{n-1} = L - L = 0 = \lim_{n\to\infty} a_n$

$\therefore$ If $\sum_{k=1}^{\infty} a_k$ converges, then $\lim_{n\to\infty} a_n = 0$

Now Write the Contrapositive Statement of the above the theorem:

If $\lim_{n\to\infty} a_n \neq 0$ **, then** $\sum_{k=1}^{\infty} a_n$ diverges.

This is called the Divergence Test.

Now let's practice a few problems:

Example 1) Show $\sum_{k=1}^{\infty} \frac{k-300}{4k+750}$ diverges.

$a_k = \frac{k-300}{4k+750}$

Divergence Test: $\lim_{n\to\infty} \frac{k-300}{4k+750} = \frac{1}{4}$

$\therefore$ Since $\lim_{k\to\infty} \frac{k-300}{4k+750} = \frac{1}{4} \neq 0$, $\sum_{k=1}^{\infty} \frac{k-300}{4k+750}$ diverges.

Determine if the following series converges or diverges. Support your answer.

(a) $\sum_{k=1}^{\infty} \frac{4n+5}{3n-1}$

$a_n = \frac{4n+5}{3n-1}$

Divergence test:

$\lim_{n\to\infty} \frac{4n+5}{3n-1} = \frac{4}{3}$

$\therefore$ Since $\lim_{n\to\infty} \frac{4n+5}{3n-1} = \frac{4}{3} \neq 0$, then $\sum_{n=1}^{\infty} \frac{4n+5}{3n-1}$ diverges

(b) $\sum_{k=1}^{\infty} \ln(\frac{2n+1}{n})$

$a_n = \ln\left(\frac{2n+1}{n}\right)$

Divergence test:

$\ln \lim_{n\to\infty} \left(\frac{2n+1}{n}\right) = \ln 2$.

$\therefore$ Since $\lim_{n\to\infty} \ln\left(\frac{2n+1}{n}\right) = \ln 2 \neq 0$, then $\sum_{n=1}^{\infty} \ln\frac{2n+1}{n}$ diverges

A Geometric Series for a repeating Decimal

Write the repeated decimal $0.3\overline{24}$ as a geometric series and then the ratio of two integers.

$0.3\overline{24} = 0.3 + \underbrace{0.024 + 0.00024 + 0.0000024 \cdots}$

G Series $r = \frac{1}{100}$ $a_1 = \frac{24}{1000} = \frac{3}{125}$

$S = \frac{3}{10} + \frac{\frac{3}{125}}{1-\frac{1}{100}} = \frac{3}{10} + \left(\frac{3}{125}\right)\left(\frac{100}{99}\right)$

$= \boxed{\frac{107}{330}}$