SUPERVISOR'S REPORT

The supervisor should complete the report below and then give this cover, enclosing the final version of the extended essay, to the Diploma Programme coordinator. The supervisor must sign this report; otherwise the extended essay will not be assessed and may be returned to the school.

Name of supervisor (CAPITAL letters)

Comments

If appropriate, please comment on the candidate's performance, the context in which the candidate undertook the research for the extended essay, any difficulties encountered and how these were overcome. These comments can help the examiner award a level for criterion H. Do not comment on any adverse personal circumstances that may have affected the candidate.

I have read the final version of the extended essay that will be submitted to the examiner.

To the best of my knowledge, the extended essay is the authentic work of the candidate.

I spent 2 hours with the candidate discussing the progress of the extended essay.

Supervisor's signature:

ASSESSMENT FORM (for examiner use only)

Candidate session number	

ACHIEVEMENT LEVEL

General assessment criteria
Refer to the general guidelines.

	Criterion	First examiner	maximum	Second examiner
A	Research question		2	
B	Approach		3	
C	Analysis/interpretation		4	
D	Argument/evaluation		4	
E	Conclusion		2	
F	Abstract		2	
G	Formal presentation		3	
H	Holistic judgement		4	

Subject assessment criteria
Refer to the subject guidelines. Not all of the following criteria will apply to all subjects; use only the criteria which apply to the subject of the extended essay.

	Criterion	First examiner	maximum	Second examiner
J				
K				
L				
M				

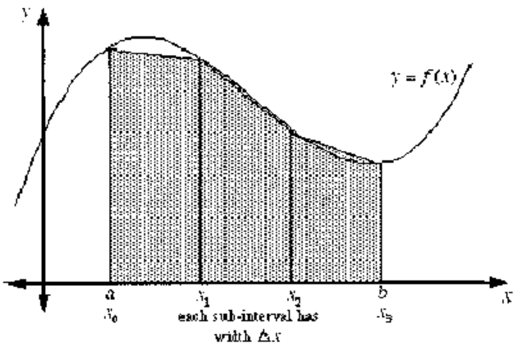
TOTAL OUT OF 36

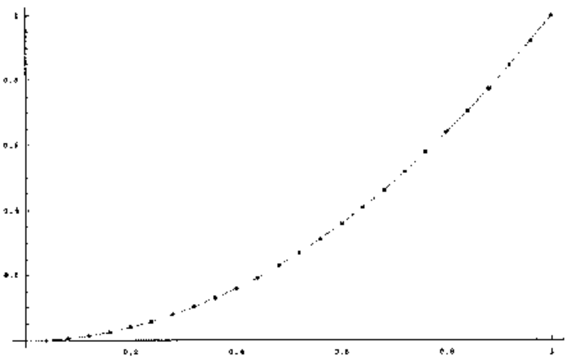
Name of first examiner (CAPITAL letters): ________ Examiner number:

Name of second examiner (CAPITAL letters): ________ Examiner number:

Extended Essay

Is using the Trapezoid Rule better than using the Midpoint Riemann Sum in finding the area under the curve in order to find the total distance?





CONTENTS

Abstract:

The aim of my extended essay is to use the Trapezoid Rule and the Midpoint Riemann Sum in order to find which is more accurate. I will begin by giving a brief background of each method and how answers are achieved by using these particular methods. I will also explain how these methods relate to each other. I will then show the mathematical process that needs to be followed in order to show which method would be more precise. The first method shows the Trapezoid Rule and its process in finding the answer for the definite integral of the function. The second method is the Midpoint Riemann Sum and the process of finding the answer for finding the approximate area under the curve for th function. This will then tell me which answer is more closely to the exact answer which was found on the calculator. The results of the essay found that the Midpoint Riemann Sum was more accurate than the Trapezoid Rule in finding the definite integral.

Research Question:

Is using the Trapezoid Rule better than using the Midpoint Riemann Sum in finding the area under the curve in order to find the total distance?

Background Information:

The Midpoint Rule uses a Riemann sum where the subinterval representatives are the midpoints of the subintervals (Trapezium Rule). The trapezium rule works by approximating the region under the graph of the function f(x) by a trapezium and calculating its area. The trapezium rule is one of a family of formulas for numerical integration called Newton-Cotes formulas. The Riemann Sums are another, often more accurate, member of the same family.

Introduction:

The Trapezoid Rule and the Midpoint Riemann Sum is used for finding the approximate area under a definite integral. This question is worthy of studying because there are many ways of finding the area under a definite integral and there needs to be evidence of which one is more accurate than the other that way one knows which answer will be more precise. In order to learn which one is better, we must understand how the process works. In a simple activity, the definite integral can be understood. By using a TI-89 and a motion detector, there is a program that explains the process of a definite integral and how to find the area under it using the Riemann Sums. After this activity is conducted, the conclusion that is made is that the more subintervals that are used, the more accurate the answer is going to be. After learning the process of definite integrals, I learned different approaches to find the area. After researching all of these ways, I found the closest was between the Trapezoid Rule and the Midpoint Riemann Sum.

Body:

In order to find which is most accurate, I will use the graph of one function and twenty-five subintervals. I will then use the process of the Trapezoid Rule and the Midpoint Riemann Sum and find out which one is more accurate than the other. I will compare the answers to the exact answer of the definite integral.

Mathematical Processes:

f(x)= 1 + x^2 on [0, 10]

The graph of this function is shown in Appendix A.

Trapezoid Rule:

$$\int_a^b f x dx = (b-a)/2n\ [f(a) + 2f(b1) + 2f(b2) \ldots + f(bn)]$$

$$\int_0^{10} f x dx\text{; use n=25}$$

Width: (10-0)/25 = 2/5

Intervals (Number Line):

0	2/5	4/5	6/5	8/5	2	12/5	14/5	16/5	18/5	4	22/5	24/5	26/5	28/5	6	32/5

34/5	36/5	38/5	8	42/5	44/5	46/5	48/5	10

Formula: Area ≈ 2/10 [f(0) + 2f(2/5) + 2f(4/5) + 2f(6/5) + 2f(8/5) + 2f(2) + 2f(12/5) + 2f(14/5) + 2f(16/5) + 2f(18/5) + 2f(4) + 2f(22/5) + 2f(24/5) + 2f(26/5) + 2f(28/5) + 2f(6) + 2f(32/5) + 2f(34/5) + 2f(36/5) + 2f(38/5) + 2f(8) + 2f(42/5) + 2f(44/5) + 2f(46/5) + 2f(48/5) + f(10)]

≈ 343.6

The graph of this function with the twenty-five subintervals is shown in Appendix B.

Now that I have found the approximate value using the Trapezoid Rule, now I will use the Midpoint Riemann Sum to approximate the next value.

Midpoint Riemann Sum:

f(x)= 1 + x^2 [0,10]

Width: (10-0)/25= 2/5
Intervals: [0,2/5] [2/5,4/5] [4/5,6/5] [6/5,8/5] [8/5,2] [2,12/5] [12/5,14/5] [14/5,16/5] [16/5, 18/5] [18/5,4] [4,22/5] [22/5,24/5] [24/5,26/5] [26/5,28/5] [28/5,6] [6,32/5] [32/5,34/5] [34/5,36/5] [36/5,38/5] [38/5,8] [8,42/5] [42/5,44/5] [44/5,46/5] [46/5,48/5] [48/5,10]

Formula: M4≈ 2/5[f(1/5) + f(3/5) + f(1) + f(7/5) + f(9/5) + f(11/5) + f(13/5) + f(15/5) + f(17/5) + f(19/5) + f(21/5) + f(23/5) + f(25/5) + f(27/5) + f(29/5) + f(31/5) + f(33/5) + f(35/5) + f(37/5) + f(39/5) + f(41/5) + f(43/5) + f(45/5) + f(47/5) + f(49/5)]
≈ 343.2

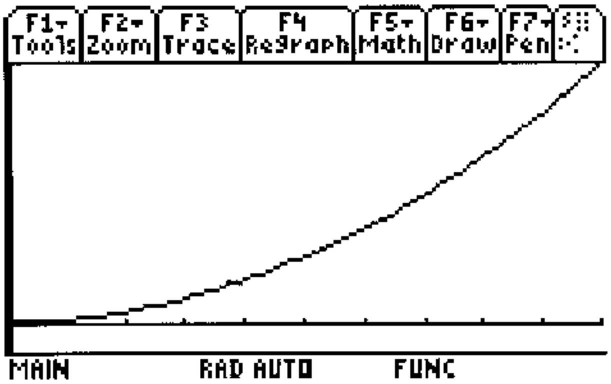
The graph of this function with twenty-five subintervals is shown in Appendix C. Now using the calculator, I will go under the graph screen, type in the function and the window from [0,10]. I will then go under F5 Math and go to number the definite integral sign and type in the lower and upper limit. The exact answer is 343.3 using the calculator.

Conclusion:

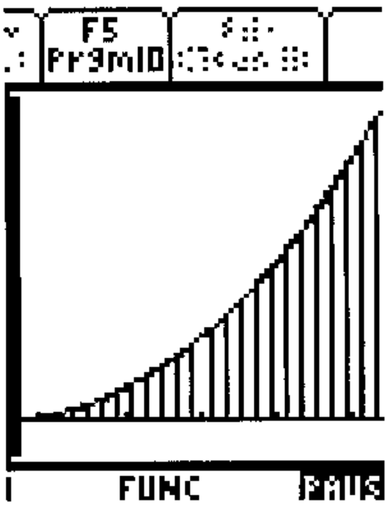
After completing this mathematical process, I can conclude that by using the calculator, the exact area of the definite integral came out to be 343.3, and the answer from the Midpoint Riemann Sum is 343.2 and the answer from the Trapezoid Rule is 343.6. Therefore, the Midpoint Riemann Sum is closer to the exact answer than the Trapezoid Rule. From this information, I can infer that the Midpoint Riemann Sum is more accurate than the Trapezoid Rule.

Appendix:

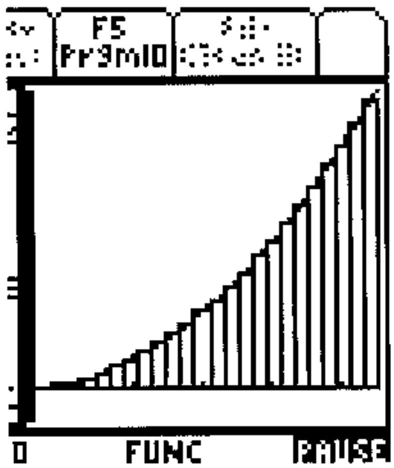
A:



B:



C:



Work Cited

"Trapezium Rule." Wikipedia. 22 Feb. 2007. Wikipedia. 26 Feb. 2007 <http://en.wikipedia.org/wiki/Trapezium_rule>.