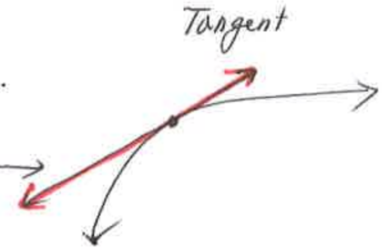
Key ①

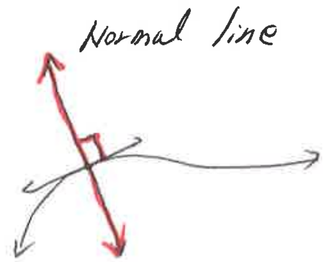
IB Math HL1: Equation(s) of the tangent line notes:



Finding the equation of a TANGENT line to the graph of f(x) at given point x=c.

$$y - f(c) = f'(c)(x - c)$$

Finding the equation of A NORMAL line to the graph of f() at given point x=c.



$$y - f(c) = -\frac{1}{f'(c)}(x - c)$$

Find the equation(s) of the tangent line at given point.

1. $f(x) = x^3(2-3x)^2$ at x=1 $\Rightarrow \left(x = 1 \quad y = (1)^3(2-3)^2 = 1 \right)$

$$\frac{df}{dx} = 3x^2(2-3x)^2 + (x^3)\cdot 2(-3)(2-3x)$$

$$\left.\frac{df}{dx}\right|_{x=1} = (3)(2-3)^2 + (1)^3\cdot 2(-3)(2-3)$$

$$= 3 + 6 = 9$$

$\Rightarrow$ $\boxed{y - 1 = 9(x-1)}$ The tangent line.

2. $f(x) = \dfrac{x^2-1}{2x+3}$ at x=0 $\Rightarrow \left(x = 0 , \quad y = \dfrac{0-1}{0+3} = \dfrac{-1}{3} \right)$

$$\frac{df}{dx} = \frac{2x(2x+3) - (x^2-1)(2)}{(2x+3)^2}$$

$$\left.\frac{df}{dx}\right|_{x=0} = \frac{0(2\cdot 0+3) - (0-1)(2)}{3^2} = \frac{2}{9}. \text{ (Slope of tangent)}$$

$\Rightarrow$ $\boxed{y + \frac{1}{3} = \frac{2}{9}(x - 0)}$ $\Rightarrow$ $\boxed{y = \frac{2}{9}x - \frac{1}{3}}$

Steps.:

① Find x & y coordinates. $(1, 1)$

② Find $\frac{df}{dx}$.

③ Calculate $\left.\frac{df}{dx}\right|_{x=1}$

④ $\Rightarrow$ put into $\boxed{y - y_1 = m(x - x_1)}$

3. Find the equation(s) of the normal line to the graph of $f(x)=\sqrt{5x}$ at x=5.

$\boxed{x=5, \quad y=\sqrt{5\cdot 5}=5}$

$\frac{df}{dx}=\sqrt{5}\cdot\frac{1}{2}\cdot x^{\frac{-1}{2}}=\frac{\sqrt{5}}{2\sqrt{x}}$

$\left.\frac{df}{dx}\right|_{x=5}=\frac{\sqrt{5}}{2\sqrt{5}}=\frac{1}{2}$. (Slope of tangent)

$\Rightarrow$ Slope of normal $\Rightarrow -2$.

$\Rightarrow \boxed{y-5=-2(x-5)}$

Practice: Derivatives using chain rule and equation(s) of the tangent line:

Find the x-coordinate of each point where the graph of the given function has a horizontal tangent line.

1. $f(x)=x\sqrt{1-3x}$

$\frac{df}{dx}=\sqrt{1-3x}+x\cdot\frac{1}{2\sqrt{1-3x}}(-3)$

$=\sqrt{1-3x}-\frac{3x}{2\sqrt{1-3x}}=0$

$=\frac{2(1-3x)-3x}{2\sqrt{1-3x}}=0$

$\Rightarrow 2-6x-3x=0 \Rightarrow \boxed{x=\frac{2}{9}}$

2. $f(x)=x^2(2x+3)^2$

$\frac{df}{dx}=2x(2x+3)^2+x^2(2)(2)(2x+3)=0$

$\Rightarrow 2x(2x+3)[2x+3+2x]=0$

$\boxed{x=0 \quad x=\frac{-3}{2} \quad x=\frac{-3}{4}}$

3. $f(x)=(2x^2-7)^3$

$\frac{df}{dx}=3(2x^2-7)^2(4x)=0$

$=12x(2x^2-7)^2=0$

$x=0 \quad 2x^2=7$

$x=\pm\sqrt{\frac{7}{2}}$

4. $f(x)=x^2e^{1-3x}$

$\frac{df}{dx}=2x\cdot e^{1-3x}+x^2(-3)\cdot e^{1-3x}=0$

$xe^{1-3x}[2-3x]=0$

$\boxed{x=0, \quad x=\frac{2}{3}}$