$y^4 = y''''$

IB Math HL1: Equation(s) of the tangent/normal line notes:

$y' = \frac{dy}{dx}$

$y'' = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$

Warm up: Find $\frac{d^2y}{dx^2}$ for the following implicit relations:

a) $x^3 + 2xy = 4 \Rightarrow$

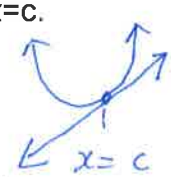
b) $\sin x + 1 = \cos y$

Solutions are attached.

Finding the equation of a TANGENT line to the graph of f(x) at given point x=c.

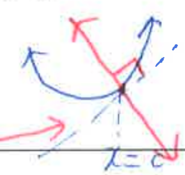
$$y - f(c) = f'(c)(x - c)$$

$y - y_1 = m(x - x_1)$



Finding the equation of A NORMAL line to the graph of f() at given point x=c.

$$y - f(c) = -\frac{1}{f'(c)}(x - c)$$



Example1) Find the equation(s) of the tangent line and normal line to the graph of $f(x) = \sqrt{5x}$ at x=5.

$= \sqrt{5} \cdot x^{\frac{1}{2}}$

1) Find $\frac{df}{dx} \Rightarrow \sqrt{5} \cdot \frac{1}{2} x^{-\frac{1}{2}} \Rightarrow \left.\frac{df}{dx}\right|_{x=5} = \frac{\cancel{\sqrt{5}}}{2\cancel{\sqrt{5}}} = \frac{1}{2}$.

2) Find $y \Rightarrow f(5) = \sqrt{5 \cdot 5} = 5$

when $x=5$

$\therefore$ 1) Tangent line: $(5,5)$ $m = \frac{1}{2}$.

$\boxed{y - 5 = \frac{1}{2}(x - 5)}$

2) Normal line: $(5,5)$ $m = -2$

$\boxed{y - 5 = -2(x - 5)}$

a) $\frac{d}{dx}(x^3+2xy)=\frac{d}{dx}(4)$

$\Rightarrow 3x^2+2y+2x\cdot\boxed{\frac{dy}{dx}}=0$

$\Rightarrow \boxed{\frac{dy}{dx}=\frac{-3x^2-2y}{2x}}$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{\left(-6x-2\frac{dy}{dx}\right)(\cancel{2}x)-(\cancel{2})(-3x^2-2y)}{\cancel{4}x^2}$

2

$=\frac{\cancel{(-6x^2-2x\frac{dy}{dx})+3x^2}+2y}{2x^2}$

$=\frac{-3x^2-\cancel{2x}\left(\frac{-3x^2-2y}{\cancel{2x}}\right)+2y}{2x^2}$

$=\frac{-\cancel{3x^2}+\cancel{3x^2}+2y+2y}{2x^2}=\boxed{\frac{2y}{x^2}}$

b) $\frac{d}{dx}(\sin x+1)=\frac{d}{dx}(\cos y)$.

$\Rightarrow \cos x=-\sin y\cdot\frac{dy}{dx}$.

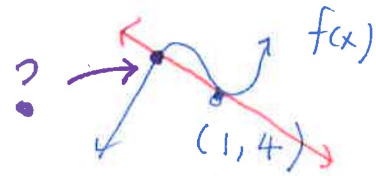
$\Rightarrow \frac{dy}{dx}=-\frac{\cos x}{\sin y}$.

$\Rightarrow \frac{d^2y}{dx^2}=\frac{\sin x\cdot\sin y+\cos x\cdot\cos y\frac{dy}{dx}}{\sin^2 y}$.

$=\frac{\sin y\left(\sin x\cdot\sin y+\cos x\cdot\cos y\cdot\left(\frac{-\cos x}{\sin y}\right)\right)}{\sin y\cdot(\sin^2 y)}$.

$=\boxed{\frac{\sin x\cdot\sin^2 y-\cos^2 x\cdot\cos y}{\sin^3 y}}$.

Example 2) Find the coordinates of the point(s) where the tangent to $f(x)=x^3+x+2$ at (1, 4) meets the curve again.



1) Find the Equation of the tangent.

$f'(x)=3x^2+1 \Rightarrow f'(1)=3+1=4.$

$y=4(x-1)+4=4x.$

2) Find the Intersection f(x) and the tangent line.

$\Rightarrow 4x=x^3+x+2 \Rightarrow x^3+x+2-4x=0 \Rightarrow x^3-3x+2=0$

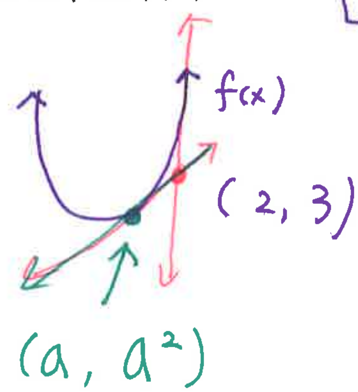
$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array} \Rightarrow (x-1)(x^2+x-2)=0$$

$(x-1)(x-1)(x+2)=0 \Rightarrow \therefore x=-2.$

$y=(4)(-2)=-8$

$\boxed{(-2,-8)}$

Example 3) Find the equations of the tangent to $f(x)=x^2$ from the external point (2, 3).



1) (a,a^2) on $f(x)=x^2 \Rightarrow f'(x)=2x$

$f'(a)=2a$

Find tangent line equation at (a,a^2)

$\Rightarrow y-a^2=2a(x-a)$

2) $y-a^2=2a(x-a)$ must pass $(2,3)$

$3-a^2=2a(2-a) \Rightarrow 3-a^2=4a-2a^2$

$\Rightarrow a^2-4a+3=0$

$(a-3)(a-1)=0$

$\Rightarrow$ $a=3$, $a=1$

plug into $y-a^2=2a(x-a)$.

3) when $a=1 \Rightarrow \boxed{y-1=2(x-1)}$

$a=3 \Rightarrow \boxed{y-9=6(x-3)}$