Math HL1
Euler's Method

"OY-ler"

Name: ____________________ Period: ___________

- **Numerical** analysis of a differential equation $y' = F(x,y)$
- Use $y' = F(x,y)$ to determine the slope of the solution at a starting point (x_0, y_0).
- Use $y_{n+1} = y_n + h \times F(x_n, y_n);\ x_{n+1} = x_n + h$ to find each subsequent point.

$h = dx$ (step size)

Euler's method is to find an approximation to a particular solution by a numerical analysis, $\boxed{y_{n+1} = y_n + h \times F(x_n, y_n)}$ $\boxed{x_{n+1} = x_n + h}$ for a given differential equation.

$\boxed{y_{n+1} = y_n + y' \cdot dx}$ $h = dx$

Example) Given $y' = \dfrac{dy}{dx} = 0.3y$, and $y(1) = 2$.

a. Use Euler's method with step size of 0.5 to find an approximation value for $y(4)$. Give your answer in 4 decimal places.

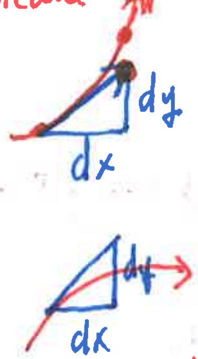
Initial values $x = 4$

n	0	1	2	3	4	5	6	7	8	9	10
x_n	1	1.5	2	2.5	3	3.5	4	X	X	X	X
y_n	2	2.3	2.645	3.0418	3.4980	4.0227	4.6261				

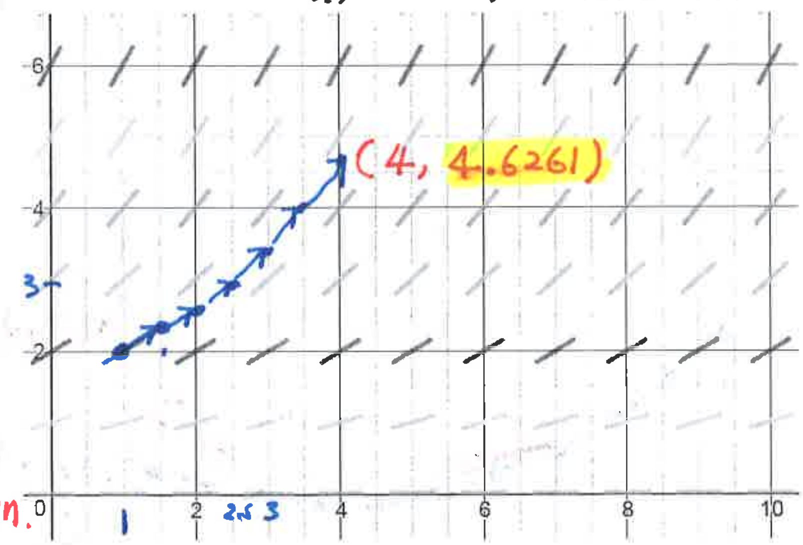
b. In the figure show the slope filed for this differential equation. Plot the y-values from (a) on the slope field. Connect the points with line segments.

$\dfrac{dy}{dx} = 0.3y \Rightarrow dy = (0.3y)dx$ $\qquad y_{n+1} = y_n + \boxed{y' \cdot dx}$

concave up.



concave down.



$y_0 = 2$

$y_1 = 2 + (0.3)(2)(0.5)$

≈ 2.3

$y_2 = 2.3 + (0.3)(2.3)(0.5)$

≈ 2.645

$y_3 = 2.645 + (0.3)(2.645) \cdot (0.5)$

c. Solve the differential equation algebraically. Find the particular solution. If $x = 4$, how well does the value of y by Euler's method agree with the actual value? Discuss the result why this approximation is greater or smaller than the true value of y.

$\dfrac{dy}{dx} = 0.3y$.

$\rightarrow dy = 0.3y \cdot dx$

$\rightarrow \dfrac{dy}{0.3y} = dx \leftrightarrow \int \dfrac{dy}{y} = \int 0.3\,dx$

$\rightarrow \ln y = 0.3x + C$

$y = e^{0.3x+C} = A \cdot e^{0.3x}$. $\Leftarrow x = 1$ $y = 2$.

$2 = A \cdot e^{0.3}$

$A = \dfrac{2}{e^{0.3}}$

$y = \left(\dfrac{2}{e^{0.3}}\right) e^{0.3x}$

$y(4) = \left(\dfrac{2}{e^{0.3}}\right)\left(e^{0.3(4)}\right) \approx \boxed{4.9192}$

1. Use Euler's Method to approximate the particular solution of the differential equation $y' = x - y$ passing through the point $(0,1)$ on the domain $[0,1]$. Use a step of $h = 0.1$ $= dx$

n	0	1	2	3	4	5	6	7	8	9	10
x_n	0	.1	.2	.3	.4	0.5	0.6	0.7	0.8	.9	1
y_n	1	0.9	.82	.758	.712	.681	.663	.657	.661	.675	.697

$$y_{n+1} = y_n + (y')(0.1)$$

$$y_1 = 1 + (0-1)(0.1) = 0.9.$$

2. For the differential equation in #1, verify that the exact solution is $y = x - 1 + 2e^{-x}$. Compare this exact solution with the approximate solution obtained in #1 by plotting.

Algebraic solution work:

$$\frac{dy}{dx} = x - y. \Rightarrow dy = (x-y) \cdot dx.$$

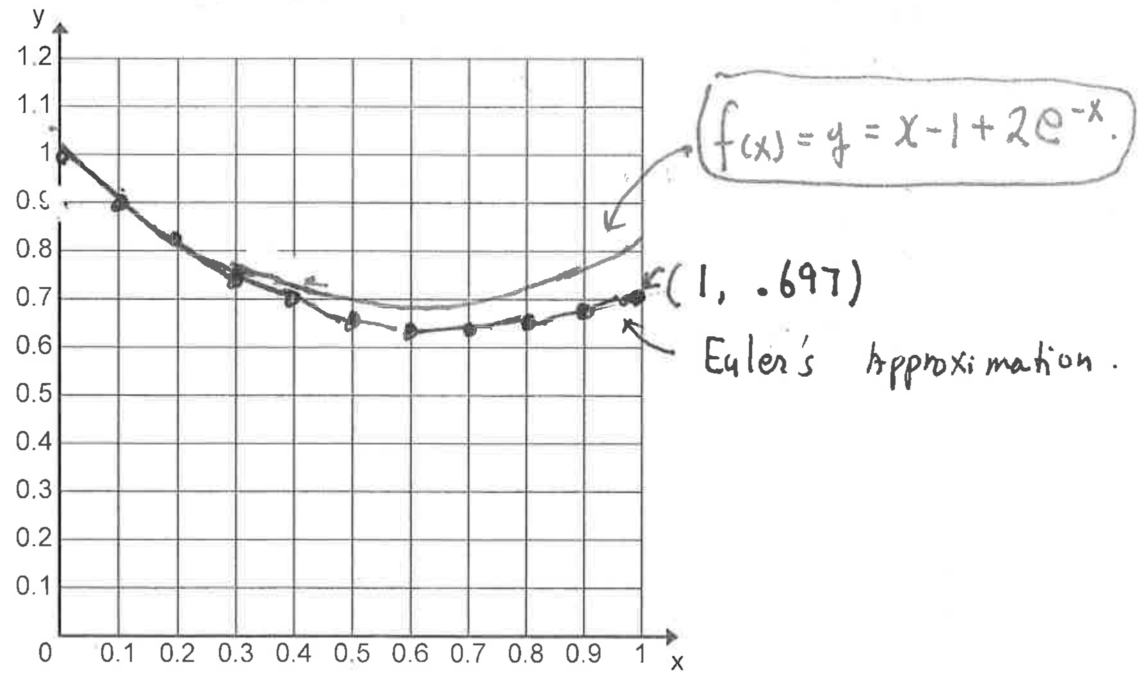
$$y' = 1 + (-2 \cdot e^{-x})$$

$$= 1 - 2e^{-x}.$$

$$y' = 1 - 2e^{-x} = x - [x - 1 + 2e^{-x}]$$

$$= 1 - 2e^{-x}$$

QED.



2. Consider the differential equation $\frac{dy}{dx}+\frac{xy}{4-x^2}=1$, where $|x|<1$ and $y = 1$ when x = 0.

$dy = \left[1-\left(\frac{xy}{4-x^2}\right)\right]dx$

Use Euler's method with h = 0.25, to find an approximate value of y when X = 1, giving your answer to two decimal places.

$dx = 0.25$

x	y	dy
0	1	0.25
0.25	1.25	.2301
0.5	1.4801	.2006
0.75	1.6807	.15832
1	1.8390	
	$\boxed{\approx 1.84}$	

$dy = \left(\frac{y+2}{xy+1}\right)dx$

3. Consider the differential equation $\frac{dy}{dx}=\frac{y+2}{xy+1}$ and y=1 when x=0.

Use Euler's method with interval h = 0.25 to find an approximate value of y when x = 1.

$(xy+1)\frac{dy}{dx} = (y+2)dx$

$= 0.25$

x	y	dy
0	1	.75
0.25	1.75	.6521
0.5	2.4021	.5000
0.75	2.9021	.3858
1	3.2879	
	$\boxed{\approx 3.29}$	

$x = 1 \quad y = 3.2879 \quad dx = 0.25$

$(xy+1)dy = (y+2)dx$