Exam (Friday)

Information.

- Limit (L'Hopital's Rule)
- Improper Integral.

Today.

- partial fraction. ⇒ Integration
- Linear D.E
- Homogeneous D.E.

Tomorrow

Thursday.

- Euler Method.
- ~~Logistic D.E~~
- ~~Dilution D.E~~

7 ~ 8 problems.

Exam on Friday.

- $\lim_{x\to\infty} \left(1+\frac{1}{2x}\right)^{3x}.$ $\quad (1+\frac{1}{\infty})^{\infty}$

$\Rightarrow \lim_{x\to\infty} \ln\left(1+\frac{1}{2x}\right)^{3x} = \ln \boxed{L}. \Rightarrow L\,?$

$\Rightarrow \lim_{x\to\infty} 3x \ln\left(1+\frac{1}{2x}\right) = \ln L$

$\left(\frac{1}{2x}\right)'$

$=\left(\frac{1}{2}\cdot x^{-1}\right)'$

$=\frac{1}{2}\cdot -1\cdot x^{-2}$

$\Rightarrow \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{2x}\right)^{0}}{\frac{1}{3x}} \left(\frac{0}{0}\right) = \ln L$

L'Hopital $\Rightarrow \lim_{x\to\infty} \frac{\left(\frac{1}{1+\frac{1}{2x}}\right)\left(\frac{1}{2}\cdot\cancel{\frac{-1}{x^2}}\right)}{\left(\frac{1}{3}\cdot\cancel{\frac{-1}{x^2}}\right)} = \ln L$

$= \lim_{x\to\infty} \frac{\left(\frac{1}{1+\cancel{\frac{1}{2x}}}\right)\cdot\left(\frac{1}{2}\right)}{\frac{1}{3}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \boxed{\frac{3}{2}} = \ln L$

$$\boxed{L = e^{\frac{3}{2}}}$$

- $\int_0^{\infty} \frac{1}{1+x^2}\,dx.$

$\Rightarrow \lim_{a\to\infty} \int_0^a \frac{1}{1+x^2}\,dx = \lim_{a\to\infty} \operatorname{atan} x \Big|_{x=0}^{x=a}$

$= \lim_{a\to\infty} \left(\operatorname{atan} a - \operatorname{atan} 0\right) = \frac{\pi}{2}$

#2 (b)

Review 3

- $\int_0^{\infty} x^2 e^{-x}\,dx$.

u	du
x^2	e^{-x}
$2x$	$-e^{-x}$
2	e^{-x}
0	$-e^{-x}$

$\rightarrow \lim_{a\to\infty} \int_0^a x^2 e^{-x}\,dx$.

$$= \lim_{a\to\infty} \left[-\frac{x^2}{e^x} - \frac{2x}{e^x} - \frac{2}{e^x} \right]_{x=0}^{x=a}$$

$$= \lim_{a\to\infty} \left[\frac{-a^2}{e^a}\left(\frac{\infty}{\infty}\right) - \frac{2a}{e^a}\left(\frac{\infty}{\infty}\right) - \cancel{\frac{2}{e^a}}^{\,0} + \left(+\cancel{\frac{2}{e^0}}^{\,2}\right) \right]$$

L'Hopital

$$= \lim_{a\to\infty} \left[\frac{-2a}{e^a}\left(\frac{\infty}{\infty}\right) \cancel{\frac{2}{e^a}}^{\,0} - 0 + 2 \right]$$

L'Hopital

$$= \lim_{a\to\infty} \left[\cancel{\frac{-2}{e^a}}^{\,0} - 0 - 0 + 2 \right] = \boxed{2}$$

Review ③

3 (d)

$$f'(x) = g(x) \quad \text{and} \quad g'(x) = f(x)$$

$$f(x) = \frac{e^{x}+e^{-x}}{2}.$$

$$\Rightarrow \int_0^{\infty} \frac{g(x)}{[f(x)]^2}\,dx$$

$\Rightarrow f'(x) = g(x)$

$u = f(x)$

$du = f'(x)\,dx$

$du = g(x)\,dx$

$x \to \infty,\ x \to 0$

$u \to \infty,\ u \to 1$

$$= \int_1^{\infty} \frac{du}{u^2}$$

$$= \lim_{a\to\infty} \int_1^{a} \frac{du}{u^2}$$

$$= \lim_{a\to\infty} \left[\frac{-1}{u}\right]_{u=1}^{u=a} = \lim_{a\to\infty}\left[\frac{-1}{a}+1\right] = \boxed{1}$$

#4 (b)

$$\int_0^{\infty} e^{-x}\cos x\,dx = \int_0^{\infty} e^{-x}\sin x\,dx$$

$$\lim_{a\to\infty}\int_0^a e^{-x}\cos x\,dx$$

$$= \lim_{a\to\infty}\left[\frac{\sin x}{2e^x} - \frac{\cos x}{2e^x}\right]_{x=0}^{x=a}$$

$$= \lim_{a\to\infty}\left[\frac{\sin a}{2e^a}\nearrow^{0} - \frac{\cos a}{2e^a}\nearrow^{0} + \frac{1}{2}\right]$$

$$= \boxed{\frac{1}{2}}$$

$$\int e^{-x}\cos x\,dx$$

u	du
e^{-x}	$\oplus$ $\cos x$
$-e^{-x}$	$\sin x$
e^{-x}	$\ominus$ $\cos x$

$\oplus$

$$\int e^{-x}\cos x\,dx = e^{-x}\sin x - e^{-x}\cos x - \int e^{-x}\cos x\,dx$$

$$\frac{\cancel{2}\int e^{-x}\cos x\,dx}{\cancel{2}} = \frac{1}{2}e^{-x}\sin x - \frac{1}{2}e^{-x}\cos x + C$$

$$\lim_{a\to\infty}\sin a = \boxed{-1 \le \sin\infty \le +1}$$

Limit does Not Exist