IB Math 2 Exit Slip Name: Key Period: __ **Graphing**

Calculator allowed. Show work. Give exact answers or round to 3 significant figures.

1. On the Northwest coast, the depth of water at time t hours after midnight is given by $d(t) = 9.3 + 6.8\cos(0.507t)$ meters.

a. What is the depth of water at 8:00 am?

$t = 8$. $d(8) = 9.3 + 6.8\cos((0.507)(8)) = 5.15$ m.

b. What is the rate of change in the depth of water at 8:00 am?

$\frac{d(t)}{dt} = (-0.507)(6.8)\sin(0.507 t) \Rightarrow t = 8 \Rightarrow \frac{dd}{dt}\Big|_{t=8} = (-0.507)(6.8)\cdot\sin(0.507\cdot 8)$

$= \boxed{2.73 \ \frac{m}{h}}$

c. What time period of a day is the tide falling? Support your answer with sign diagram.

$\frac{dd}{dt} = 0 \Rightarrow -3.4\sin(0.507t) = 0$

$0 = \sin(0.507t) \Rightarrow 0, \pi, 2\pi, 3\pi.$

$t = 0,\ 6.20,\ 12.4$

$0 \quad \ominus \quad 6.20 \quad \oplus \quad 12.4 \quad \ominus \quad 18.6 \quad \oplus$

$\Rightarrow$ falling $(0, 6.2) \cup (12.4, 18$

period: 12 hrs

2. A particle moving on the x-axis has position after an elapsed time of t seconds: $s(t) = t^3 - 9t^2 + 24t$.

a. Find the velocity of the particle at time t.

$v(t) = 3t^2 - 18t + 24$

b. Find the acceleration at time t.

$a(t) = 6t - 18 = 0 \Rightarrow t = 3$

c. Find the position of the particle at the times when it reverses direction.

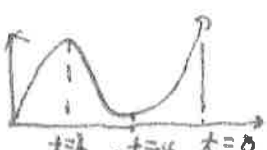
$3t^2 - 18t + 24 = 0 \qquad 3(t^2 - 6t + 8) = 0 \Rightarrow 3(t-2)(t-4) = 0$

$= 2, \ = 4$

$\boxed{t = 2,\ t = 4}$

$S(2) \qquad S(4)$

d. What is the total distance traveled by the particle during the first 8 seconds?



Total distance $= |S(0) - S(2)| + |S(2) - S(4)| + |S(4) - S(8)|$

$= |0 - 20| + |20 - 16| + |16 - 128| = \boxed{136 \text{ units}}$

e. At what times is the particle's speed increasing and decreasing.

$t=0 \quad t=2 \quad t=4$

$v(t)$: $0 \quad \oplus \quad \ominus \quad \oplus$

$t=3$

$a(t)$: $\ominus \quad \oplus$

Speed Increasing: $(2, 3) \cup (4, \infty)$

speed decreasing: $(0, 2) \cup (3, 4)$

key ②

3, Determine whether the Mean Value Theorem can be applied to $f(x)=\frac{1}{x}$ in the interval $[1,4]$. If the Mean value Theorem is applied, find all values of c in the given interval.

$f(x)=\frac{1}{x}$ is continuous and differentiable in $[1,4] \Rightarrow \therefore$ MVT is applicable.

MVT: $f'(c)=\frac{f(b)-f(a)}{b-a} \Rightarrow f(4)=\frac{1}{4}, \; f(1)=1, \; f'(c)=\frac{-1}{c^2}$

$\frac{-1}{c^2}=\frac{\frac{1}{4}-1}{4-1} \Rightarrow \frac{-1}{c^2}=\frac{\frac{-3}{4}}{3} \Rightarrow \frac{-1}{c^2}=\frac{-\cancel{3}}{\cancel{3}\cdot 4} \Rightarrow c^2=4$

$c=\pm 2 \Rightarrow \boxed{c=2}$

(−2 is out range)

4. Given $f(x)=2x+|x|$, prove that f is continuous but not differentiable at the point (0, 0)

$\Rightarrow f(x)=\begin{cases} 3x & x\geq 0 \\ x & x<0 \end{cases}$

Continuity: $\lim_{x\to 0^+} 3x = \lim_{x\to 0^-} x \Rightarrow 0=0 \quad \therefore f(x)$ is continuous

Differentiability: $\lim_{x\to 0^+} 3 \neq \lim_{x\to 0^-} 1 \Rightarrow 3\neq 1 \quad \therefore f(x)$ is Not differentiable.

5. Given $f(x)=\begin{cases} e^{-x^2}(-x^3+2x^2+x) & x\leq 1 \\ ax+b & x>1 \end{cases}$:

a) Find the exact values of a and b if f is continuous and differentiable at x=1

To be continuous: $\lim_{x\to 1^-} e^{-x^2}(-x^3+2x^2+x)=\lim_{x\to 1^+} ax+b \Rightarrow \boxed{\frac{2}{e}=a+b}$

To be differentiable: $\lim_{x\to 1^-}\left[-2xe^{-x^2}(-x^3+2x^2+x)+e^{-x^2}(-3x^2+4x+1)\right]=\lim_{x\to 1^+} a$

$a=\frac{-2}{e}$ $\quad \boxed{b=\frac{4}{e}}$

b) Use "Rolle's theorem applied to f, to prove $2x^4-4x^3-5x^2+4x+1=0$ has a root in the interval (-1, 1).

say $g(x)$

$\Rightarrow x\leq 1 \Rightarrow$ use $e^{-x^2}(-x^3+2x^2+x)$

$[-1,1]$

$f(-1)=e^{-1}(1+2-1)=\frac{2}{e}$

$f(1)=e^{-1}(-1+2+1)=\frac{2}{e}$

$\therefore$ The Rolle's theorem tells you $f'(x)=0$ exists On $[-1,1]$

$\Rightarrow$ Hence $f'(x)$ is Quartic and has a root $(-1,1)$.

Notes:
$g(x)=f'(x)$
$=(2x^4-4x^3-5x^2+4x+1)e^{-x^2}$

c) Hence, $2x^4-4x^3-5x^2+4x+1=0$ has at least two roots in the interval (-1, 1)

$g(x)=2x^4-4x^3-5x^2+4x+1$

$g(-1)=2-4-5+4+1=-2<0$

$g(1)=1-4-5+4+1=-2<0$

$g(0)=1>0$

$\Rightarrow \therefore$ There are at least two roots.

Notes: $f'(x)=-2xe^{-x^2}(-x^3+2x^2+x)+e^{-x^2}(-3x^2+4x+1)=e^{-x^2}\left[2x^4-4x^3-5x^2+4x+1\right]$