Exit slip #1.

#1. Ratio test.

$$\lim_{n\to\infty}\left[\frac{(-1)^{n+1}(x-2)^{n+1}}{(n+1)(2)^{n+1}}\right]\cdot\left[\frac{n\cdot 2^n}{(-1)^n(x-2)^n}\right]$$

$$=\lim_{n\to\infty}\left[\frac{\cancel{(-1)^n}\cdot(-1)(x-\cancel{2)^n}(x-2)}{(n+1)(\cancel{2)^n}\cdot 2}\right]\cdot\left[\frac{n\cdot\cancel{2^n}}{\cancel{(-1)^n}(\cancel{x-2})^n}\right]$$

$$=\lim_{n\to\infty}(-1)\left(\frac{n(x-2)}{n+1}\right)\cdot\frac{1}{2}=\frac{(x-2)}{2}(-1)$$

$$\left|\frac{x-2}{2}\right|<1 \Rightarrow -2<x-2<2 \Rightarrow 0<x<4$$

a) $\boxed{R=2}$

#2 Ratio test.

$$\lim_{n\to\infty}\left[\frac{(-1)^{n+1}(x)^{2(n+1)}}{(2(n+1)+1)}\right]\left[\frac{2n+1}{(-1)^n x^{2n}}\right]$$

$$=\lim_{n\to\infty}\left(\frac{\cancel{(-1)^n}(-1)(\cancel{x^{2n}})x^2}{(2\cancel{n+3})\ 1}\right)\left(\frac{\cancel{2n+1}\ 1}{\cancel{(-1)^n}\cdot\cancel{x^{2n}}}\right)=(-1)(x^2)\Rightarrow |x^2|<1.$$

a) $R=1$

#3. Ratio test.

$$\lim_{n\to\infty}\left(\frac{(-1)^{n+1}\cdot x^{2(n+1)}}{(n+1)!}\right)\left(\frac{n!}{(-1)^n\cdot x^{2n}}\right)$$

$$=\lim_{n\to\infty}\left(\frac{\cancel{(-1)^n}(-1)\,\cancel{x^{2n}}\cdot x^2}{\cancel{n!}\,(n+1)}\right)\left(\frac{\cancel{n!}}{\cancel{(-1)^n}\cdot\cancel{x^{2n}}}\right)=\lim_{n\to\infty}\frac{(-1)\cdot x^2}{n+1}=0.$$

a) $R=\infty$.

#4. $\displaystyle\sum_{n=1}^{\infty}\frac{n!\,x^n}{(2n)!}$

$$\lim_{n\to\infty}\left[\frac{(n+1)!\,x^{n+1}}{(2(n+1))!}\right]\left[\frac{(2n)!}{n!\,x^n}\right]$$

$$=\lim_{n\to\infty}\left[\frac{\cancel{n!}\,(n+1)\cdot\cancel{x^n}\cdot x}{\overset{2n+2}{}(\cancel{2n!})(2n+1)(2n+2)}\right]\left[\frac{\cancel{(2n)!}}{\cancel{n!}\,\cancel{x^n}}\right]=\lim_{n\to\infty}\frac{x\,(n+1)}{(2n+1)(2n+2)}$$

$=0$

a) $R=\infty$.

#1. Interval of convergence.

b) $\sum \frac{(-1)^n(-2)^n}{n\cdot(2)^n} = \sum \frac{(-1)^n(-1)^n\cdot \cancel{2^n}}{n\cdot \cancel{2^n}} = \sum \frac{1}{n}.$

$x=0$

Diverges by p series $p=1$.

$x=4$ $\sum \frac{(-1)^n \cancel{(2^n)}}{n\cdot \cancel{(2)^n}} = \sum \frac{(-1)^n}{n} \Rightarrow$ Alt. Series conditional convergence.

$\lim_{n\to\infty} \frac{1}{n} = 0$ $\left(\frac{1}{n}\right)' = \frac{-1}{n^2} < 0$ decreasing.

$\Rightarrow (0, 4]$.

#2. b)

$\left.\begin{matrix} x=1 \\ x=-1 \end{matrix}\right) \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1} \Rightarrow$ Alt. Series. $\Rightarrow$ Conditional convergence.

$\lim_{n\to\infty} \frac{1}{2n+1} = 0.$ $\left(\frac{1}{2n+1}\right)' = \frac{-2}{(2n+1)} < 0$ decreasing.

$\Rightarrow (-1, 1)$

#3. #4. b) Interval of convergence.

$(-\infty, \infty)$

④

2.

$f(x) = e^{\frac{x}{4}}$

a) $f(0) = 1$

$f'(0) = \frac{1}{4}$.

$f''(0) = \frac{1}{16}$.

$f'''(0) = \frac{1}{64}$.

$f^{4}(0) = \frac{1}{4^4}$

a)

$\Rightarrow P_4(x) = 1 + \frac{1}{4}x + \frac{1}{4^2}\frac{x^2}{2!} + \frac{1}{4^3}\frac{x^3}{3!} + \frac{1}{4^4}\frac{x^4}{4!}$

b) $\sum_{n=0}^{\infty} \frac{x^n}{4^n(n!)}$.

3.

① $f(x) = \tan x = \frac{\sin x}{\cos x}$. $\Rightarrow f(0) = 0$.

② $f'(x) = \frac{\cos^2 x - \sin^2 x}{\cos^2 x} \Rightarrow f'(0) = 1$

$= \frac{\cos^2 x - (1 - \cos^2 x)}{\cos^2 x} = \frac{2\cos^2 x - 1}{\cos^2 x}$. $= 2 + \frac{1}{\cos^2 x} = 2 + (\cos x)^{-2}$

③ $f''(x) = +2(\cos x)^{-3} \cdot (+\sin x)$ $\quad f''(0) = 0$.

④ $f'''(x) = -6((\cos x)^{-4}(-\sin x) + 2(\cos x)^{-3}(+\cos x)$

$f'''(0) = +2$.

$\Rightarrow P_3(x) = 0 + x + \frac{0x^2}{2!} + \frac{2x^3}{3!}$

$= \boxed{x + \frac{x^3}{3}}$

#4. $n=3$. $c=3$

a. $f(x) = \frac{2}{x+1}$ $\quad f(3) = \frac{2}{4} = \frac{1}{2}$.

$n=0$

$f'(x) = \frac{-2}{(x+1)^2}$ $\quad f'(3) = \frac{-2}{(4)^2} = \frac{-1}{8}$

$n=1$

$f''(x) = \frac{+4}{(x+1)^3}$ $\quad f''(3) = \frac{+4}{(4)^3} = \frac{+1}{16}$

$n=2$

$f'''(x) = \frac{-12}{(x+1)^4}$ $\quad f'''(3) = \frac{-12}{(4)^4} = \frac{-3}{16}$.

$n=3$

$$\Rightarrow T_3(x) = \frac{1}{2} - \frac{1}{8}\cdot(x-3) + \frac{1}{16}\frac{(x-3)^2}{2!} - \frac{3}{16}\frac{(x-3)^3}{3!}$$

$n=4$ $\quad c=5$

b. $f(x) = \ln x$ $\quad f(5) = \ln 5$

$n=0$

$f'(x) = \frac{1}{x}$. $\quad f'(5) = \frac{1}{5}$.

$n=1$

$n=2$. $f''(x) = \frac{-1}{x^2}$ $\quad f''(5) = \frac{-1}{5^2}$

$n=3$ $f'''(x) = \frac{2}{x^3}$ $\quad f'''(5) = \frac{2}{5^3} = \frac{2}{5^3}$.

$n=4$

$f^4(x) = \frac{-6}{x^4}$ $\quad f^4(5) = \frac{-6}{5^4}$

$$T_3(x) = \ln 5 + \frac{1}{5}(x-5) - \frac{1}{5^2}\frac{(x-5)^2}{2!} + \frac{2}{5^3}\frac{(x-5)^3}{3!} - \frac{6}{5^4}\frac{(x-5)}{4!}$$

#4

$n=5 \qquad c=\pi.$

c.

$n=0 \quad f(x)=\cos(x). \quad f(\pi)=\cos(\pi)=-1$

$n=1 \quad f'(x)=-\sin(x). \quad f'(\pi)=0$

$n=2 \quad f''(x)=-\cos(x). \quad f''(\pi)=1$

$n=3 \quad f'''(x)=+\sin(x). \quad f'''(\pi)=0$

$n=4 \quad f^4(x)=+\cos(x) \quad f^4(\pi)=-1$

$n=5 \quad f^5(x)=-\sin(x) \quad f^5(\pi)=0.$

$$\Rightarrow T_5(x)=-1+0(x-\pi)+\frac{(x-\pi)^2}{2!}+\frac{0(x-\pi)^3}{3!}-\frac{(x-\pi)^4}{4!}$$

$$+\frac{0(x-\pi)^5}{5!}$$

$$=-1+\frac{(x-\pi)^2}{2}-\frac{(x-\pi)^4}{24}$$

#5.

(a) $a_{n+1} \Rightarrow \left|\frac{(-1)^{n+1}}{(2(n+1))!}\right| < 0.001$

$$\frac{1}{(2n+2)!} < 0.001$$

$$(2n+2)! > \frac{1}{0.001}$$

$$(2n+2)! > 1000.$$

$n=1$ $4! \Rightarrow 24$

$n=2$ $6! \Rightarrow 720$

$n=3$ $8! \Rightarrow 40320$ $\boxed{n=3}$

(b) $a_{n+1} \Rightarrow \left|\frac{(-1)^{n+1}(n+1)^2}{2^{n+1}}\right|$

$$\frac{(n+1)^2}{2 \cdot 2^n} < 0.001$$

$$\frac{2 \cdot 2^n}{(n+1)^2} > 1000$$

$$\frac{2^n}{(n+1)^2} > 500$$

$n=1$

$n=2.$

$n=3.$

$\boxed{n=18} \Rightarrow 726 > 500$