Exit Slip # 2 Solutions.

#1. $\displaystyle\int \frac{3}{x^2+3x-4}\,dx = \int \frac{3}{(x+4)(x-1)}\,dx$

$A(x-1)+B(x+4) = 3$

$x=1 \Rightarrow 5 \cdot B = 3$

$B = \frac{3}{5}$

$x=-4 \Rightarrow A = -\frac{3}{5}$

$\displaystyle = \int \frac{A}{x+4}\,dx + \int \frac{B}{x-1}\,dx$

$\displaystyle = \int \frac{\frac{-3}{5}}{x+4}\,dx + \int \frac{\frac{3}{5}}{x-1}\,dx$

$\displaystyle = \frac{3}{5}\ln(x-1) - \frac{3}{5}\ln(x+4) + C$ OR $\displaystyle \ln \frac{(x-1)^{3/5}}{(x+4)^{3/5}} + C$

#2. $\displaystyle\int \frac{5}{(x^2+1)(x+1)}\,dx$

$\displaystyle = \int \frac{Ax+B}{x^2+1}\,dx + \int \frac{C}{x+1}\,dx$

$(Ax+B)(x+1) + C(x^2+1) = 5$

$x=-1$ $\quad C = \frac{5}{2}$

$x=0$ $\quad B(1) + C = 5 \Rightarrow B = 5 - \frac{5}{2} = \frac{5}{2}.$

$x=1$ $\quad (A+\frac{5}{2})(2) + \frac{5}{2}(2) = 5 \Rightarrow 2A + 5 + \cancel{5} = \cancel{5}$

$A = \frac{-5}{2}.$

$\displaystyle = \frac{5}{2}\left[\int \frac{-x+1}{x^2+1}\,dx + \frac{1}{x+1}\,dx\right]$

$\displaystyle = \frac{5}{2}\left[-\frac{1}{2}\ln(x^2+1) + \ln(x+1) + \arctan(x)\right] + C$

$\displaystyle = \frac{5}{2}\left[\ln\frac{(x+1)}{\sqrt{x^2+1}} + \arctan(x)\right] + C.$

#2. a.

$$\int (1+\tan^2 x)\,dx$$

$$= \int (1 + \sec^2 x - 1)\,dx$$

$$= \int \sec^2 x \, dx$$

$$= \boxed{\tan x + c}$$

b.

$$\int \sin^2 x \, dx$$

$$= \int \tfrac{1}{2}(1 - \cos 2x)\,dx$$

$$= \tfrac{1}{2}\left(x - \tfrac{1}{2}\sin 2x\right) + c$$

$$= \tfrac{1}{2}\left(x - \sin x \cos x\right) + c$$

#3.

$$\int \frac{e^x}{e^{2x} + 6e^x + 13}\,dx$$

$$= \int \frac{e^x}{(e^{2x} + 6e^x + 9) + 4}\,dx$$

$$= \int \frac{e^x}{(e^x+3)^2 + 4} \qquad \begin{pmatrix} u = e^x + 3 \\ du = e^x dx \end{pmatrix}$$

$$= \int \frac{1}{u^2+4}\,du$$

$$= \frac{1}{2}\arctan\left(\frac{e^x}{2}\right) + c$$

#4

c. $u = x - \frac{1}{2}$ $\Rightarrow$ $\dfrac{1}{4(x-\frac{1}{2})^2+4} = \dfrac{1}{4}\left[\dfrac{1}{u^2+1}\right]$

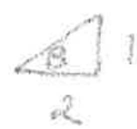
$du = dx$.

$\Rightarrow \int f(x)\,dx = \frac{1}{4}\int \frac{1}{u^2+1}\,du$

$= \frac{1}{4}\left[\tan^{-1} u\right] + C$ $\left(\begin{matrix} x=3.5 & u=5 \\ x=1 & u=\frac{1}{2}\end{matrix}\right)$

d. $\displaystyle\int_{\frac{1}{2}}^{3} f(x)\,dx = \frac{1}{4}\left[\tan^{-1}(3) - \tan^{-1}\left(\frac{1}{2}\right)\right]$

$\tan A = 3$ $\quad\tan B = \frac{1}{2}$.



$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$.

$\frac{1}{4}\tan^{-1}\left(\dfrac{3-\frac{1}{2}}{1+3\cdot\frac{1}{2}}\right) = \frac{1}{4}\tan^{-1}(1)$

$= \frac{1}{4}\cdot\frac{\pi}{4} = \boxed{\frac{\pi}{16}}$