IB Pre HL Exit Slip #2 Name: Key

Calculators is allowed!! Show all your work!

1. Solve for x if $0 \le x \le 2\pi$

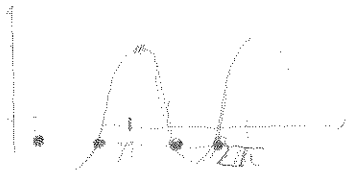
a. $\sqrt{2}\cos 2(x-\pi) = -1$

$\theta = 2(x-\pi)$

$0 \le x \le 2\pi$

$-\pi \le x-\pi \le \pi$

$-2\pi \le \theta \le 2\pi$



$\cos 2(x-\pi) = -\frac{1}{\sqrt{2}}$

$2(x-\pi) = A$

$\cos A = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$2(x-\pi) = \frac{3\pi}{4}$

$x-\pi = \frac{3\pi}{8}$

$x = \frac{11\pi}{8}$

$2(x-\pi) = -\frac{3\pi}{4}$

$x-\pi = -\frac{3\pi}{8}$

$x = -\frac{3\pi}{8} + \pi = \frac{5\pi}{8}$

b. $9 - 12\sin x = 4\cos^2 x$

$9 - 12\sin x = 4(1-\sin^2 x)$

$9 - 12\sin x = 4 - 4\sin^2 x$

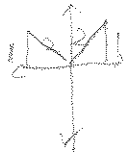
$4\sin^2 x - 12\sin x + 5 = 0$

$2\sin x \quad -1$

$2\sin x \quad -5$

$(2\sin x - 1)(2\sin x - 5) = 0$

$\sin x = \frac{1}{2} \qquad \sin x = \frac{5}{2}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$2(x-\pi) = \frac{11\pi}{4}$

$x = \frac{11\pi}{8} + \frac{2\pi}{8} = \frac{13\pi}{8}$

$2(x-\pi) = \frac{5\pi}{4}$

$x = \frac{5\pi}{8} + \frac{2\pi}{8} = \frac{13\pi}{8}$

$2(x-\pi) = -\frac{5\pi}{4}$

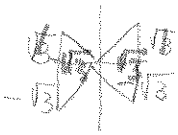
$x - \pi = -\frac{5\pi}{8}$

$x = -\frac{5\pi}{8} + \frac{8\pi}{8} = \frac{3\pi}{8}$

c. $\sqrt{3}\tan^2 x = 1 \Rightarrow 3\tan^2 x = 1$

$\tan^2 x = \frac{1}{3}$

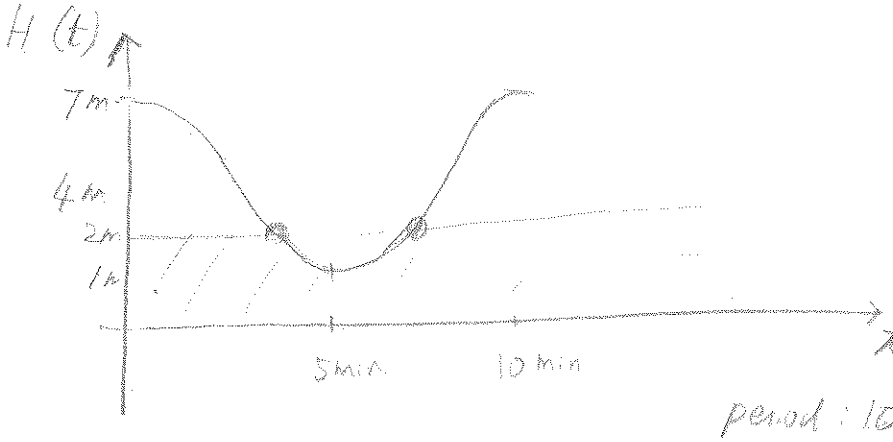
$\tan x = \pm\frac{1}{\sqrt{3}}$

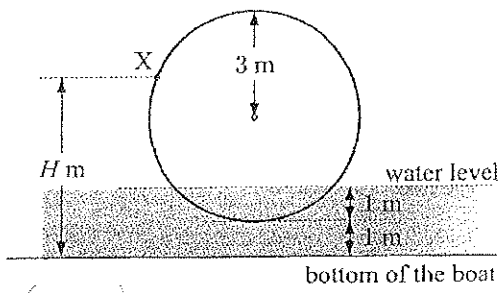


$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

2. A paint spot X on the outer rim of the wheel of a paddle-steamer. The wheel has radius 3 m. It rotates anticlockwise at a constant rate, and X is seen every 5 seconds from maximum height to minimum. H is the distance of X above the bottom of the boat. At time t=0, X is at its highest point of 7 m from the bottom of the boat.

a. Sketch the graph the position of X from the bottom of boat in the time interval of $0 \le t \le 10$.





period : 10 min
Amplitude : 3
axis : $y = 4$

b. Write a cosine model for H(t) in the form $H(t) = a\cos(b(t-h)) + k$.

$H(t) = 3\cos\left[\frac{2\pi}{10}(t)\right] + 4$

$H(t) = 3\cos\left(\frac{\pi}{5}t\right) + 4$

c. At what time t does X first enter the water?

$H = 2m$

$2 = 3\cos\frac{\pi}{5}t + 4$

$-\frac{2}{3} = \cos\frac{\pi}{5}t$

$\frac{\pi}{5}t = \cos^{-1}\left(-\frac{2}{3}\right)$

$t = \cos^{-1}\left(-\frac{2}{3}\right)\left(\frac{5}{\pi}\right)$