Exit Slip (Group Activity)

- $\frac{dy}{dx} = y^2\sqrt{x}$ (Separable) and passing through $(9, \ -\frac{1}{18})$, find a particular solution.

- $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$, find a general solution.

- $xdy - \sqrt{x^2 - y^2}\,dx = ydx$, find a general solution.,

- $\int \frac{2x^3 - 4x - 8}{(x^2+4)(x^2-x)}dx$.

♦

$$\frac{dy}{dx} = y^2\sqrt{x}$$

$$\frac{dy}{y^2} = \sqrt{x}\,dx$$

$$\frac{-1}{y} = \frac{2}{3}x^{3/2} + C \quad \Leftarrow (9, -\tfrac{1}{18})$$

$$18 = \frac{2}{3}(9)^{3/2} + C \qquad C = 0$$

$$\Rightarrow \quad \frac{-1}{y} = \frac{2}{3}x^{3/2}$$

$$\boxed{y = -\frac{3}{2}x^{-3/2}}$$

♣

$$\left(x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}\right) \div x$$

$$\left(\frac{dy}{dx} + \frac{3}{x}\cdot y = \frac{\sin x}{x^3}\right) \times x^3$$

$$\underbrace{x^3\frac{dy}{dx} + 3x^2y} = \sin x \qquad I = e^{\int \frac{3}{x}dx} = x^3$$

$$\int d[x^3 \cdot y] = \int \sin x\,dx$$

$$y = \frac{1}{x^3}\left[-\cos x + C\right]$$

$$\boxed{y = Cx^{-3} - x^{-3}\cos x}$$

♠

$$\left(x\,dy - \sqrt{x^2-y^2}\,dx = y\,dx\right) \div dx$$

$$\left(x\frac{dy}{dx} - \sqrt{x^2-y^2} = y\right) \div x$$

$$\frac{dy}{dx} - \sqrt{1 - \frac{y^2}{x^2}} = \frac{y}{x}$$

$$v = \frac{y}{x}$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\left[\cancel{v} + x\frac{dv}{dx}\right] - \sqrt{1-v^2} = \cancel{v}$$

$$\frac{x}{dx} = \frac{\sqrt{1-v^2}}{dv}$$

$$\int\frac{dx}{x} = \int\frac{dv}{\sqrt{1-v^2}}$$

$$\boxed{\ln x = \arcsin v + C}$$

$$\arcsin\left(\frac{y}{x}\right) = \ln x - C$$

$$\frac{y}{x} = \sin\left[\ln x - c\right] \quad \Rightarrow \boxed{y = x\sin\left[\ln x - c\right]}$$

♥ Answer key attached

$$\int \frac{2x^3-4x-8}{(x^2+4)(x^2-x)}\,dx = \int \frac{2x^3-4x-8}{x(x-1)(x^2+4)}\,dx$$

$$= \int \frac{A}{x}\,dx + \int \frac{B}{x-1}\,dx + \int \frac{Cx+D}{x^2+4}\,dx$$

※ Finding $A, B, C, \& D$.

$$A(x-1)(x^2+4) + Bx(x^2+4) + (Cx+D)x(x-1) = 2x^3-4x-8$$

$x=0$ $\quad A(-1)(4) = -8 \quad \boxed{A=2}$

$x=+1$ $\quad B(5) = 2-4-8 = -10 \quad \boxed{B=-2}$

$x=-1$ $\quad 2(-2)(5) + (-2)(-1)(5) + [C(-1)+D](-1)(-2) = 2(-1)^3-4x-8$

$2 = -C+D$

$x=-2$ $\quad 8 = 2C+D$

$\Rightarrow \quad C=2 \quad D=4$

$$\Rightarrow \int \frac{2}{x}\,dx + \int \frac{-2}{x-1}\,dx + \int \frac{2x+4}{x^2+4}\,dy = \int \frac{2}{x}\,dx + \int \frac{-2}{x-1}\,dx + \int \frac{2x}{x^2+4}\,dx + \int \frac{4}{x^2+4}\,dx$$

$$= \ln x - 2\ln|x-1| + \ln|x^2+4| + 2\arctan\left(\frac{x}{2}\right) + C.$$

$$= \ln\left[\frac{x(x^2+4)}{(x-1)^2}\right] + 2\arctan\left(\frac{x}{2}\right) + C$$