Chapter 20 (Exit slip Solution.) Optimization.

#1. a. Horizontal Asymp: $y = 3$

Vertical Asympt: $x = 1$. $\Rightarrow$ (1, 3) Intersection

$\Rightarrow x = 1$.

b. $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $\Leftarrow$ Distance Formula.

$$= \sqrt{(x-1)^2 + \left(\frac{3x}{x-1} - 3\right)^2}$$

$$= \sqrt{(x-1)^2 + \left[\frac{3x - 3(x-1)}{x-1}\right]^2}$$

$$= \sqrt{(x-1)^2 + \left(\frac{3}{(x-1)}\right)^2}$$

c. $d^2 = (x-1)^2 + \left(\frac{3}{x-1}\right)^2$ $\Leftarrow$ optimize to find min. point.

$$= (x-1)^2 + 3^2 (x-1)^{-2}$$

$$\frac{d[d^2]}{dx} = 2(x-1) + 3^2(-2)(x-1)^{-3} = 0$$

$$= 2(x-1) - \frac{18}{(x-1)^3} = 0$$

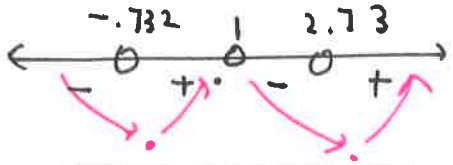
$$\Rightarrow \frac{1}{2}(x-1) = \frac{18 \cdot 9}{(x-1)^3}$$

$$\Rightarrow (x-1)^4 = 9$$

$$\Rightarrow x = \pm\sqrt[4]{9} + 1$$

$x = 2.73$ or $x = -.732$

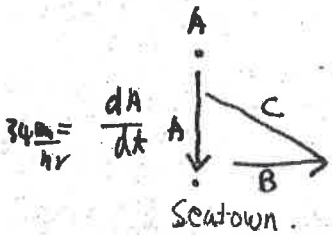
$\Rightarrow$ First derivative Sign diagram. $\frac{d\,d^2}{dx}$.



$\Rightarrow$ (2.73, 4.73)

(−.732, 1.27)

#2



$\frac{dB}{dt} = \frac{27 km}{hr}$

$\frac{dc}{dt} = ?$

$A = 68 km.$

$B = (3)(27) = 81$

$C = \sqrt{68^2 + 81^2} = 105.76$

$A^2 + B^2 = C^2$

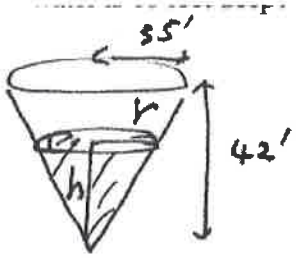
$2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2C \cdot \frac{dc}{dt}$

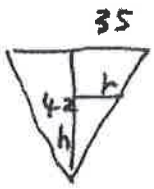
$\Rightarrow (68)(-34) + (81)(27) = 105.76 \frac{dc}{dt}$

Solve for $\frac{dc}{dt}$

$= -1.18 \frac{km}{hr}$

#3.





$\frac{dV}{dt} = 56 \frac{ft^3}{min}$

$h = 16 ft$

$\frac{35}{42} = \frac{r}{h}$

$\frac{dA}{dt} = ?$

$35h = 42r$

$h = \frac{42}{35} r$

$= \frac{6}{5} r$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 \left(\frac{6}{5} r\right)$

$V = \frac{6\pi}{15} r^3$

$\frac{dV}{dt} = \frac{6\pi}{15} 3r^2 \frac{dr}{dt}$

$56 = \frac{6\pi}{15}(3)\left(\frac{40}{3}\right)^2 \cdot \frac{dr}{dt}$

$\frac{dr}{dt} = .083556$

$r = \frac{40}{3}$

$A = \pi r^2$

$\frac{dA}{dt} = \pi(2r)\frac{dr}{dt}$

$\frac{dA}{dt} = (\pi)(2)\left(\frac{40}{3}\right) = 7 \frac{ft}{min}$

(0.083556)