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1. A particle is moving along a line so that is velocity is $v(t)=t^3-10t^2+29t-20$ feet per second at time t.

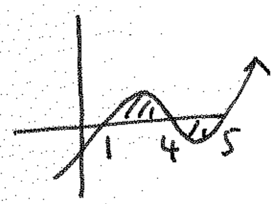
   a. Determine the speed is increasing ~~and~~ or decreasing when t = 3. Show your reasoning.

$$v(t) = t^3 - 10t^2 + 29t - 20 \Rightarrow v(3) = 4$$
$$a(t) = 3t^2 - 20t + 29 \Rightarrow a(3) = -4$$
$\Rightarrow$ decreasing

   b. What is the displacement (net) of the particle on the time interval [1, 5].

$$\int_1^5 (t^3 - 10t^2 + 29t - 20)\,dt = \frac{1}{4}t^4 - \frac{10}{3}t^3 + \frac{29}{2}t^2 - 20t \Big]_{t=1}^{t=5}$$
$$= \left[\frac{1}{4}(5)^4 - \frac{10}{3}(5)^3 + \frac{29}{2}(5)^2 - 20(5)\right] - \left[\frac{1}{4} - \frac{10}{3} + \frac{29}{2} - 20\right] = 10.\overline{6}$$

   c. What is the total distance traveled by the particle on the time interval [1, 5].



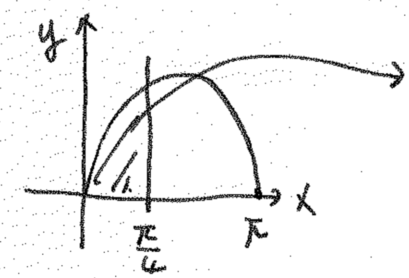
$$\Rightarrow \int_1^4 (t^3 - 10t^2 + 29t - 20)\,dt - \int_4^5 (t^3 - 10t^2 + 29t - 20)\,dt$$
$$= \left[\frac{1}{4}t^4 - \frac{10}{3}t^3 + \frac{29}{2}t^2 - 20t\right]_{t=1}^{t=4} - \left(\frac{1}{4}t^4 - \frac{10}{3}t^3 + \frac{29}{2}t^2 - 20t\right)_{t=4}^{t=5}$$
$$11.25 - (-.58\overline{3}) = 11.8\overline{3} \text{ OR } \frac{71}{6}$$

2. Given $f''(t)=5\sin 3t+\cos(\frac{1}{2}t)$, $f(0)=2$, and $f'(0)=-1$, Find f(x).

$$f'(t) = -\frac{5}{3}\cos 3t + 2\sin\left(\tfrac{1}{2}t\right) + C$$
$$-1 = -\frac{5}{3}(\cos 0) + 2\sin(0) + C \quad \Leftarrow f'(0) = -1$$
$$C = -1 + \frac{5}{3} \qquad C = \frac{2}{3}$$
$$f(x) = -\frac{5}{9}\sin 3t - 4\cos\left(\tfrac{1}{2}t\right) + \frac{2}{3}t + D \quad \Leftarrow f(0) = 2$$
$$2 = -4 + D \qquad D = 6$$
$$\Rightarrow f(x) = -\frac{5}{9}\sin 3t - 4\cos\left(\tfrac{1}{2}t\right) + \frac{2}{3}t + 6$$

3. Find the area of the region bounded by the given equations.

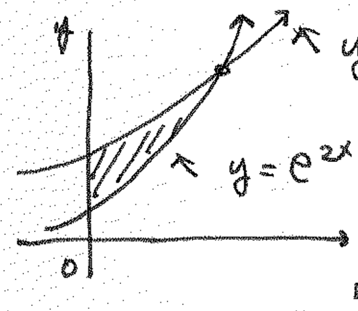
a $y = 4\sin 2x$, the x-axis, the y-axis, and $x = \frac{\pi}{4}$.



$$\int_0^{\frac{\pi}{4}} (4\sin 2x)\,dx$$

$$= -2\cos 2x \Big|_{x=0}^{x=\frac{\pi}{4}} = -2\cos\left(\cancel{2\left(\tfrac{\pi}{4}\right)}\right) + 2\cos 0 = \boxed{2}$$

b. $y = 2e^x$, $y = e^{2x}$, and $x = 0$



$$2e^x - e^{2x} = 0$$

$$e^x(2 - e^x) = 0$$

$$\cancel{e^x} \neq 0 \qquad e^x = 2 \Rightarrow x = \ln 2$$

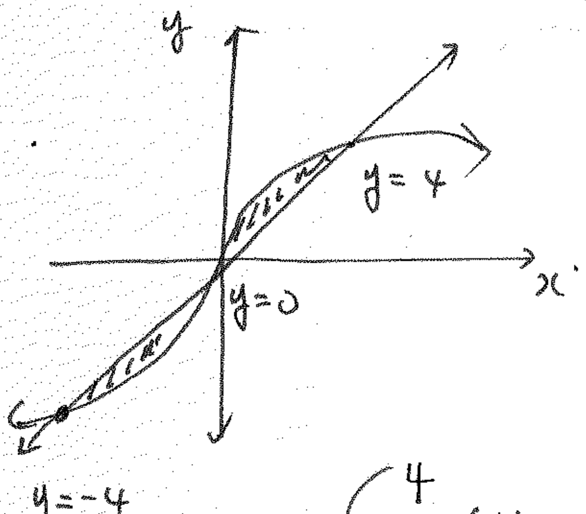
$$\int_0^{\ln 2} (2e^x - e^{2x})\,dx$$

$$= 2e^x - \tfrac{1}{2}e^{2x} \Big|_{x=0}^{x=\ln 2}$$

$$= \left[2e^{\ln 2} - \tfrac{1}{2}e^{2\ln 2}\right] - \left[2e^0 - \tfrac{1}{2}e^0\right]$$

$$= [4 - 2] - [2 - \tfrac{1}{2}] = 2 - 1.5 = \boxed{0.5}$$

c. $x = y^3$ and $x = 16y$



$$y^3 = 16y \qquad y^3 - 16y = 0$$

$$y(y+4)(y-4) = 0$$

$$y = 0,\; y = -4,\; y = 4$$

$$\int_0^4 (16y - y^3)\,dy + \int_{-4}^0 (y^3 - 16y)\,dy$$

$$= 2\int_{-4}^0 (y^3 - 16y)\,dy = 2\left[\tfrac{1}{4}y^4 - \tfrac{16y^2}{2}\right]_{y=0}^{y=4}$$

$$= 2\left[\tfrac{1}{4}(4)^4 - \tfrac{16}{2}(4)^2\right] = 128.$$