

Expectation of the random variable x:

$E(x) = 4$

$E(x) = \sum_{i=1}^n x_i p = x_1 p(X = x_1) + x_2 p(X = x_2) + x_3 p(X = x_3) \dots + x_n p(X = x_n)$

Expected value of x, E(x): mean value of x: The expected value is a measure of a probability weighted average or a long-run average.

Properties of E(x)

- 1) $E(a) = a$
- 2) $E(ax) = aE(x)$
- 3) $E(f(x)) = \sum_{i=1}^n f(x_i) p(x = x_i)$
- 4) $E(ax + b) = aE(x) + b$

Ex) For the random variable X with probability distribution defined by

x	1	2	3	4
P(X=x)	1/10	2/10	3/10	4/10

Find the mode, median, and mean values of x.

mode: 4

Median: 3

Mean ($E(x)$) = $(1)(\frac{1}{10}) + (2)(\frac{2}{10}) + (3)(\frac{3}{10}) + 4(\frac{4}{10}) = 3$

Ex) For the Probability distribution shown below

x	0	1	2	3
P(X=x)	1/6	1/2	1/5	2/15

Find

- a) $E(x)$
- b) $E(x^2)$
- c) $E(x^2 + 3x - 1)$

a) $E(x) = (0)(\frac{1}{6}) + (1)(\frac{1}{2}) + (2)(\frac{1}{5}) + 3(\frac{2}{15}) = \frac{13}{10}$

b) $E(x^2) = (0)^2(\frac{1}{6}) + (1)^2(\frac{1}{2}) + (2)^2(\frac{1}{5}) + (3)^2(\frac{2}{15}) = \frac{5}{2}$

c) $E(x^2 + 3x - 1) = E(x^2) + 3E(x) - 1 = \frac{5}{2} + 3(\frac{13}{10}) - 1 = \frac{27}{5}$

Variance (σ^2): The Average of the squared deviations about the mean μ .

$$Var(x) = E(X - \mu)^2 = \sum_{i=1}^n (x_i - \mu)^2 P(x_i)$$

Notes: σ^2 ; $Var(x) = E(x^2) - \mu^2 = E(x^2) - (E(x))^2$

Proof:

$$\begin{aligned} Var(x) &= E(X - \mu)^2 \\ &= E(X^2 - 2\mu X + \mu^2) \\ &= E(X^2) - 2\mu E(X) + \mu^2 \\ &= E(X^2) - 2\mu \cdot \mu + \mu^2 \\ &= E(X^2) - \mu^2 \end{aligned}$$

Q.E.D
 \leftarrow properties of $E(x)$
 $\leftarrow E(x) = \mu$

Standard Deviation: $\sigma = \sqrt{Var(x)}$

- Variance and Standard deviation provide a measure of the variability of the random variable.

Properties of the Variance

- 1) $Var(a) = 0$
- 2) $Var(aX) = a^2 Var(x)$
- 3) $Var(aX + b) = a^2 Var(x)$

\Rightarrow For the verification of these properties.
 Do Investigation 1 on page 793.

Ex) A random variable x has a probability distribution defined by

x	0	1	2	3	4
P(X=x)	1/16	3/16	7/16	3/16	2/16

- Find $E(x)$.
- Find $Var(4x+2)$
- Find $Sd(3-x)$

c) To Find $Sd(3-x)$
 \Rightarrow Find $Var(3-x)$
 $= (-1)^2 Var(x)$

a $E(x) = (0)(\frac{1}{16}) + (1)(\frac{3}{16}) + 2(\frac{7}{16}) + (3)(\frac{3}{16}) + (4)(\frac{2}{16})$

b) $Var(4x+2) = 4^2 Var(x)$
 (Find $E(x^2)$ and $E(x)$)
 then find $Var(x)$