①

#1.

$2x^2y + 3y^2 = 16 \quad \in (1, p)$

(a) $2p + 3p^2 = 16$

$\Rightarrow 3p^2 + 2p - 16 = 0$

$3p \qquad +8$

$p \qquad -2$

$\Rightarrow (3p+8)(p-2) = 0$

$\cancel{p = -\frac{8}{3}} \quad \boxed{p = 2} \qquad p > 0$

(b) $\left.\frac{dy}{dx}\right|_{\substack{x=1 \\ y=2}}$ $\Leftrightarrow$ Gradient of the tangent.

$2x^2 \cdot \boxed{\frac{dy}{dx}} + 4xy + 6y \cdot \boxed{\frac{dy}{dx}} = 0$

$\frac{dy}{dx}\left[2x^2 + 6y\right] = -4xy.$

$\frac{dy}{dx} = \frac{-4xy}{2x^2+6y} = \frac{-2xy}{x^2+3y} \quad \in (1, 2)$

$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{(-2)(2)}{1^2+6} = \frac{-4}{7}$

②

#3. $y = \frac{k}{x} + \ln(x^2)$

I: Normal Equation $3x + 2y = C.$

$$y = -\frac{3}{2}x + \frac{c}{2}.$$

Tangent Slope: $\frac{2}{3}$.

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{-k}{x^2} + \frac{2x}{x^2}$$

$$= \frac{-k}{x^2} + \frac{2}{x}.$$

II: $\left.\frac{dy}{dx}\right|_{x=2} = \frac{-k}{4} + \frac{2}{2}$. (Tangent Slope)

$$\Rightarrow \frac{2}{3} = \frac{-k}{4} + 1 \Rightarrow \frac{+k}{4} = +\frac{1}{3}$$

$$k = \frac{4}{3}$$

#4

$$y = \frac{1}{2}\sin 2x + \cos x \qquad 0 \le x \le 2\pi$$

$$\frac{dy}{dx} = \cancel{1}\,\cancel{\left(\tfrac{1}{2}\right)(2)}\cos 2x - \sin x = 0$$

$$\Rightarrow \cos 2x - \sin x = 0$$

$$\Rightarrow 1 - 2\sin^2 x - \sin x = 0$$

$$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$$

$$\begin{array}{ll} 2\sin x & -1 \\ \sin x & +1 \end{array}$$

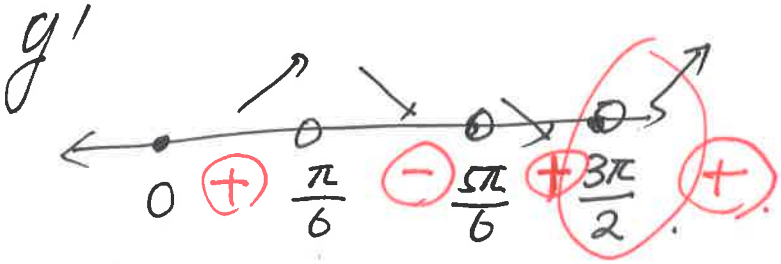
$$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$$

$$\sin x = \frac{1}{2} \qquad \sin x = -1$$

$$x = \frac{\pi}{6}, \ \frac{5\pi}{6} \qquad x = \frac{3\pi}{2}.$$

$0 \le x \le 2\pi$

Sign diagram



$y' = (2\sin x - 1)(\sin x + 1)$

Max $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{4}\right)$

Min $\left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{4}\right)$

2nd derivative Test.

$$y' = 2\sin^2 x + \sin x - 1$$

$$y'' = 4\sin x \cdot \cos x + \cos x .$$

$$y''\left(\frac{\pi}{6}\right) < 0$$ Max

$$y''\left(\frac{5\pi}{6}\right) > 0$$ Min.

#7. $f(x) = \dfrac{x^2+5x+5}{x+2} \quad x \neq -2$.

(a)

$$f'(x) = \frac{(2x+5)(x+2)-(1)(x^2+5x+5)}{(x+2)^2}$$

$$= \frac{2x^2+9x+10-x^2-5x-5}{(x+2)^2}$$

$$= \boxed{\frac{x^2+4x+5}{(x+2)^2}} = \cancel{x+5}$$

(b)

pre HL.

$$f'(x) > 2$$

$$\frac{x^2+4x+5}{(x+2)^2} - 2 > 0$$

$$\frac{x^2+4x+5-2(x+2)^2}{(x+2)^2} > 0$$

$$\frac{x^2+4x+5-2(x^2+4x+4)}{(x+2)^2} > 0$$

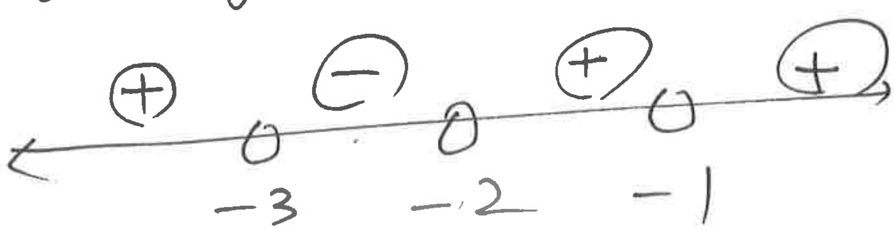
$$\frac{x^2+4x+5-2x^2-8x-8}{(x+2)^2} > 0$$

$$\Rightarrow \frac{-x^2-4x-3}{(x+2)^2} > 0$$

$$\frac{x^2+4x+3}{(x+2)^2} < 0$$

$$\frac{(x+3)(x+1)}{(x+2)^2} < 0$$

Sign diagram.



$$x \in (-3, -2)$$

#8.

$$x^2 + xy + y^2 = 19.$$

(a) $\left.\frac{dy}{dx}\right|_{x=-2}$.

$$2x + y + x \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{(-2x - y)}{[2y + x]}$$

$$\left.\frac{dy}{dx}\right|_{x=-2,\ y=5} = \frac{(-2)(-2) - 5}{(2 \cdot 5 - 2)} = \frac{-1}{8}$$

when $x = -2 \Rightarrow 4 - 2y + y^2 = 19$

$$y^2 - 2y - 15 = 0$$

-5

$+3$

$y = 5$, $y = \cancel{-3}$ $\quad y > 0$

Equation: $\boxed{y - 5 = \frac{-1}{8}(x + 2)}$

$$\frac{dy}{dx} = \frac{-2x-y}{2y+x}$$

$2y + x = 0$

Slope is undefined (parallel to the y-axis).

$x = -2y. \Rightarrow x^2 + xy + y^2 = 19$

$(-2y)^2 + (-2y)(y) + y^2 = 19.$

Solve.

#5. A (1, 2, 3) and B (3, 1, 2)

$$\vec{OA} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad \vec{OB} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

(a) $\vec{OA} \times \vec{OB} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{vmatrix}$

$= i(4-3) - j(2-9) + k(1-6)$

$= i + 7j - 5k$.

(b) Area of $\triangle OAB$

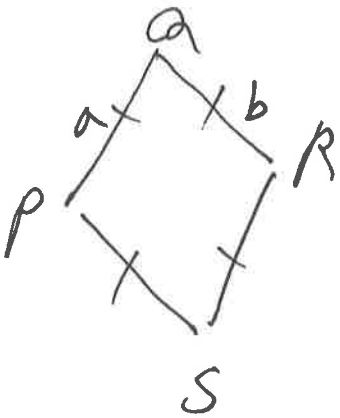
$= \frac{1}{2}\sqrt{1^2 + 49 + 25} = \frac{\sqrt{75}}{2} = \frac{5\sqrt{3}}{2}$

$\cancel{5\sqrt{75}}$

#10

PQRS is a Rhombus

$\vec{PQ} = a$ and $\vec{QR} = b$.



$|\vec{PQ}| = |\vec{QR}| = |RS| = |PS|$

$\Rightarrow |a| = |b|$ by def. of Rhombus.

$\cancel{\vec{PQ}}$ $\vec{PR} = a+b$

$\vec{QS} = b-a$ OR $-a+b$.

$\Rightarrow$ To Show $\vec{PR} \perp \vec{QS} \Rightarrow$ Show $\vec{PR} \cdot \vec{QS} = 0$

$\vec{PR} \cdot \vec{QS} = (a+b)\cdot(b-a)$

$= \cancel{ab} - a\cdot a + b\cdot b - \cancel{a\cdot b}$

$= -a^2 + b^2 = 0$ Since $a = b$.

$\therefore \vec{PR} \perp \vec{QS}$