ANSWERS

IB HL MATH 2 FIRST SEMESTER FINALS PRACTICE

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Let me know in person if you have any questions!

1. Refer to **Differentiation** in **Calculus Review**

$$\frac{dy}{dx} = 4xy, \qquad when\ x = 1\ and\ y = e$$

$$\frac{dy}{dx} = 4(1)(e) = 4e$$

This is the slope of the function at $(1, e)$
Now, plug into point slope formula $y = f'(x)(x - x_1) + y_1$

$$y = 4e(x - 1) + e$$

2. Refer to **Integration by Parts** in **Calculus Review**

$$u = x \qquad dv = \sin(x)$$

$$du = dx \quad v = -\cos(x)$$

$$\int uv' = uv - \int vdu$$

$$x(-\cos(x)) - \int -\cos(x)\, dx$$

$$-x\cos(x) + \int \cos(x)\, dx$$

3. Refer to **Integration by u-Substitution** in **Calculus Review**

$$u = (1 + x^2) \quad du = 2x, \qquad \frac{1}{2}du = x$$

$$\int_0^a \frac{1}{2}\left(\frac{du}{u}\right)$$

$$\frac{1}{2}[\ln(u)] = \frac{1}{2}[\ln(1 + x^2)]_{x\,=\,0}^{x\,=\,a}$$

$$\frac{1}{2}(\ln(1 + a^2) - \frac{1}{2}(\ln(1))$$

$$\frac{1}{2}(\ln(1 + a^2))$$

4. Refer to **Second Fundamental Theorem of Calculus** in **Calculus Review**

$$-\frac{d}{dx}(\int_7^{5^x} \frac{\ln(t)}{\sin(t)}dt$$

$$-\frac{\ln(5^x)}{\sin(5^x)} \cdot \ln(5)\,(5^x)$$

5. Refer to **Linear Differential Equations** in **Calculus Review**

Integrating factor

$$e^{-\int \cot(x)dx} = e^{-\int \frac{\cos(x)}{\sin(x)}dx}$$

$$u = \sin(x) \quad du = \cos(x)\, dx$$

$$\int \frac{du}{u} = \ln(u) = \ln(sin(x))$$

$$e^{-\ln(sin(x))} = \frac{1}{\sin(x)}$$

Continue…

$$\frac{dy}{dx} - \frac{\cos(x)}{\sin(x)} y = \sin^3(x)$$

$$\left(\frac{1}{\sin(x)}\right) \cdot \left(\frac{dy}{dx} - \frac{\cos(x)}{\sin(x)} y\right) = (\sin^3(x)) \cdot \left(\frac{1}{\sin(x)}\right)$$

$$\int d\left(y \cdot \frac{1}{\sin(x)}\right) = \int \sin^2(x)\, dx$$

$$y \cdot \frac{1}{\sin(x)} = \int \frac{1}{2}(1 - \cos(2x))dx$$

$$y = \sin(x)\left(\frac{1}{2}\left(x - \frac{1}{2}(\sin(2x)) + c\right)\right)$$

$$y = \sin(x)\left(\frac{1}{2}(x - \sin(x)\cos(x)) + c\right)$$

6. Refer to **Homogeneous Differential Equation** in **Calculus Review**

$$\left(\frac{1}{x^2}\right) \cdot x^2 \frac{dy}{dx} = (y^2 + 2xy) \cdot \left(\frac{1}{x^2}\right)$$

$$\frac{dy}{dx} = \left(\frac{y}{x}\right)^2 + 2\left(\frac{y}{x}\right)$$

Plug in the standards

$$v + x \cdot \frac{dv}{dx} = v^2 + 2v$$

$$x \cdot \frac{dv}{dx} = v^2 + v$$

$$\frac{dv}{v^2 + v} = \frac{dx}{x}$$

$$\int \frac{dv}{v^2 + v} = \int \frac{dx}{x}$$

Partial Fractions

$$\int \frac{dv}{v^2 + v} = \int \left(\frac{1}{v} - \frac{1}{v+1}\right) dv$$

$$\ln(v) - \ln(v+1)$$

Go back

$$\int \frac{dx}{x} = \int \frac{dv}{v^2 + v}$$

$$\ln(x) = \ln(v) - \ln(v+1)$$

$$\ln(x) = \ln\left(\frac{y}{x}\right) - \ln\left(\frac{y}{x} + 1\right) + c$$

7. Refer to **Integration** in **Calculus Review**
8. Refer to **Implicit Differentiation** in **Calculus Review**

Plug in $x = 1$ first to find y

$$1^3 - 3(1^2)y + (1)y^2 = -1$$
$$y^2 - 3y + 2 = 0$$
$$(y-2)(y-1) = 0$$
$$y = 2, \qquad y = 1$$

Integrate

$$3x^2 - 6xy - 3x^2\frac{dy}{dx} + (y^2 + x\left(2y\left(\frac{dy}{dx}\right)\right) = 0$$
$$\frac{dy}{dx}(-3x^2 + x2y) = 6xy - 3x^2 - y^2$$
$$\frac{dy}{dx} = \frac{6xy - 3x^2 - y^2}{2xy \ - 3x^2}$$

Plug (similar to 1)

$$\frac{dy}{dx}_{(1,2)} = \frac{12 - 3 - 4}{4 - 3} = \frac{5}{1}$$
$$y = 5(x-1) + 2$$
$$\frac{dy}{dx}_{(1,1)} = \frac{6 - 3 - 1}{2 - 3} = -2$$
$$y = -2(x-1) + 1$$

9. Refer to **How to Classify Lines with Algebra** in **Vector Applications**
10. Refer to **Cross Product** and **Planes** in **Vector Applications**