IB Math HL 1: Curve Analysis
NO CALCULATOR

Name: Key

- **The first Derivative test and intervals of increasing and decreasing**

For the given functions,
1) Find the intervals where the function is increasing and decreasing.
2) Determine the local max and/or the local min of the critical numbers (stationary points) by a sign diagram.

a. $f(x) = 3x^4 - 8x^3 + 2$

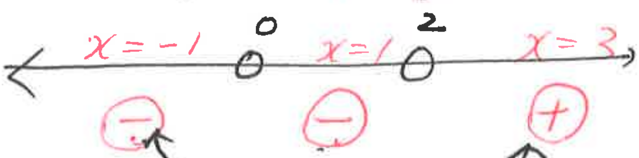
$$\frac{df}{dx} = 12x^3 - 24x^2 = 0$$

$$12x^2(x-2) = 0$$

$x = 0 \quad x = 2$: Critical points.

Sign diagram of $f'(x)$

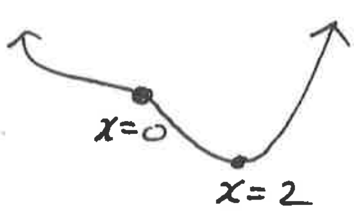
$$f'(x) = 12x^2(x-2)$$



1) Increasing : $(2, \infty)$

decreasing : $(-\infty, 0) \cup (0, 2)$

2)



NO Local Max : (None.)

Local Min : $x = 2$.

b. $f(x) = \dfrac{3x-9}{x^2 - x - 2}$

$$\frac{df}{dx} = \frac{3(x^2-x-2) - (3x-9)(2x-1)}{(x^2-x-2)^2}$$

$$= \frac{-3x^2+18x-15}{(x-2)^2(x+1)^2}$$

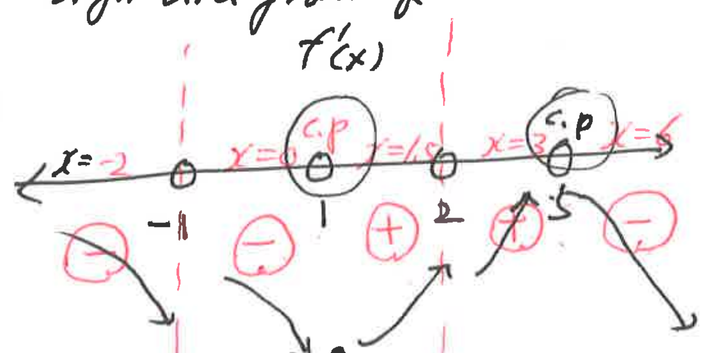
$x \neq 2 \quad x \neq -1$

$$-3x^2+18x-15 = 0$$

$$-3(x-5)(x-1) = 0$$

$x = 5 \quad x = 1$: critical points.

Sign diagram of $f'(x)$



$$f'(x) = \frac{-3(x-5)(x-1)}{(x-2)^2(x+1)^2}$$

1) Increasing : $(1, 2) \cup (2, 5)$

Decreasing : $(-\infty, -1) \cup (-1, 1)$

$\cup (5, \infty)$

2) Local Max : $x = 5$

Local Min : $x = 1$

IB Math 2 Inflection point and the intervals of concavity Name________________

For the given function

1) Find the x-coordinate where $f''(x)=0$: The point is called an inflection point at which concavity changes.
2) Find the interval where the curve is concave up and the curve is concave down: The curve is concave up If t $f''(x)>0$ and the curve is concave down if $f''(x)<0$.

a. $g(x)=-x^3+3x^2+5$

$\frac{dg}{dx}=-3x^2+6x$

$\frac{d^2g}{dx^2}=-6x+6=0$

$=-6(x-1)=0$

$x=1$

1) Inflection point

$x=1$

2) Sign diagram of $f''(x)$

$x=0$ $x=1$ $x=2$

$\oplus$ $\ominus$

$f''(x)=-6(x-1)$

concave up: $(-\infty,1)$

concave down: $(1,\infty)$

b. $f(x)=$ $\sin^2 x+\cos x$ $[0,\pi]$

$\frac{df}{dx}=2\sin x\cos x-\sin x$

$\frac{d^2f}{dx^2}=2\cos x\cdot\cos x+2\sin x(-\sin x)-\cos x=0$

$=2\cos^2 x-2\sin^2 x-\cos x=0$

$=2\cos^2 x-2(1-\cos^2 x)-\cos x=0$

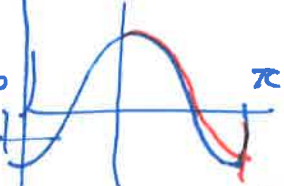
$=2\cos^2 x-2+2\cos^2 x-\cos x=0$

$=4\cos^2 x-\cos x-2=0$

$\Rightarrow 4A^2-A-2=0$

$A\approx -.593$ 0.843

$\cos x\approx -0.593$ $\cos x\approx 0.843$



$[0,\pi)$

$x=\cos^{-1}(-0.593)\approx 2.21$

$x=\cos^{-1}(0.843)\approx 0.568$

1) Inflection points:

$x=0.568$ $x=2.21$

2) concave up

$[0,0.568)\cup(2.21,\pi]$

concave down

$(0.568,\ 2.21)$

Sign diagram of $f''(x)$

$x=0.4$

0 $\oplus$ 0.568 $\ominus$ 2.21 $\oplus$ π

$f''(x)=4\cos^2 x-\cos x-2$

#5 $f(x) = (x^2-3)\cdot e^x$ $\qquad \frac{df}{dx} = 0$

$\frac{df}{dx} = (2x)\cdot e^x + e^x(x^2-3) = 0$ $\quad$ Solve for $x = ?$

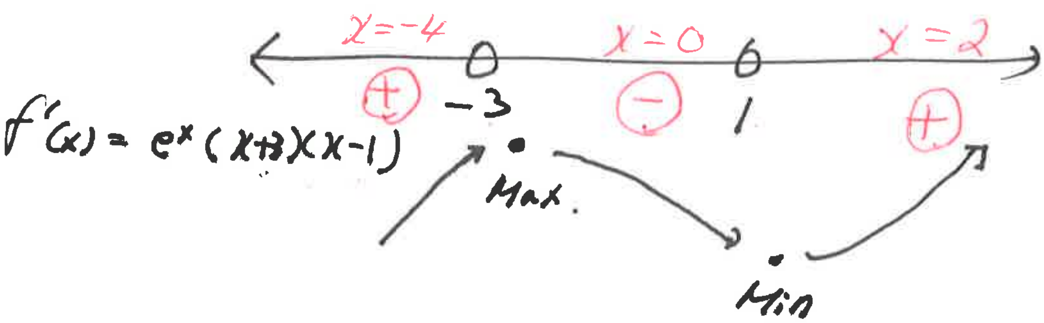
$\Rightarrow \quad e^x[2x + x^2 - 3] = 0$

$e^x[x^2 + 2x - 3] = 0$

$e^x(x+3)(x-1) = 0$

$x = -3 \quad x = 1$: critical points.

Sign diagram of $\frac{df}{dx}$.



Increasing: $(-\infty, -3) \cup (1, \infty)$

decreasing: $(-3, 1)$