IB Math 2 FTC and Indefinite Integrals Name____________________

**Riemann Sum**

$$\sum_{k=1}^{n} f(a+k\cdot\Delta x)\cdot\Delta x$$

Sum of n rectangles approximating the area under $f(x)$.

**Antiderivative**

If $F'(x)=f(x)$, then $F(x)$ is an antiderivative of $f(x)$

**Indefinite Integral**

$$\int f(x)\,dx = F(x)+C$$

Represents all antiderivatives of $f(x)$

$$\lim_{n\to\infty}\sum_{k=1}^{n} f(a+k\cdot\Delta x)\cdot\Delta x$$

Sum of infinitely many rectangles of infinitesimal width.

$\Delta x = \dfrac{b-a}{n}$

$$\int_a^b f(x)\,dx$$

The **exact area** under $f(x)$ on the interval $[a,b]$

**FTC!!!!!**

If $f(x)$ is continuous with antiderivative $F(x)$, then $\int_a^b f(x)\,dx = F(b)-F(a)$.

1. a. We want to use 8 left-hand rectangles to approximate $\int_0^4 \sqrt{16-x^2}\,dx$.

$k=0 \rightarrow k=7.$

i. Represent the sum of the 8 rectangle areas using sigma notation.

$\Delta x = \dfrac{4-0}{8} = \dfrac{1}{2}$ $\quad \displaystyle\sum_{k=0}^{7} \frac{1}{2} f\left(0+k\left(\tfrac{1}{2}\right)\right)$ where $f(x)=\sqrt{16-x^2}$

ii. Use your GFC to find the sum of the rectangle areas accurate to 4 decimal places.

$\boxed{13.3595}$

b. We want to use 100 right-hand rectangles to approximate $\int_0^4 \sqrt{16-x^2}\,dx$.

$k=1 \rightarrow k=100$

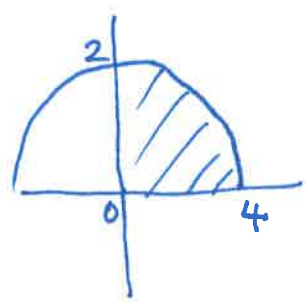
i. Represent the sum of the 100 rectangle areas using sigma notation.

$\Delta x = \dfrac{4-0}{100} = \dfrac{4}{100} = \dfrac{1}{25}$ $\quad \displaystyle\sum_{k=1}^{100} \frac{1}{25} f\left(0+k\left(\tfrac{1}{25}\right)\right)$

ii. Use your GFC to find the sum of the rectangle areas accurate to 4 decimal places.

$\boxed{12.4817}$

c. We want to use graphical methods and known facts to find $\int_0^4 \sqrt{16-x^2}\,dx$.




i. Use area formulas to find the exact value of the integral.

$\int_0^4 \sqrt{16-x^2}\,dx = \frac{1}{4}(\pi 4^2).$

ii. Write down your answer accurate to four decimal places. How does this value compare with bii?

$\approx 12.5664 \Rightarrow$ The Exact Area is less than (a) and greater than (b)

**IB Math HL1 (20 B notes: Antidifferentiation)**

Name Key Period: ________

Group warm-up

Given $f'(x)$, find $f(x)$:

| $f'(x)=\dfrac{df}{dx}$ | $f(x)$ |
|---|---|
| 1. $f'(x)=3x^2$ | $f(x)=x^3+C$ where C is a constant. |
| 2. $f'(x)=2x-4x^4$ | $f(x)=x^2-\frac{4}{5}x^5+C$ |
| 3. $f'(x)=2x^2+x$ | $f(x)=\frac{2}{3}x^3+\frac{1}{2}x^2+C$ |
| 4. $f'(x)=\frac{2}{3}x^2-2x$ | $f(x)=\frac{2}{9}x^3-x^2+C$ |
| 5. $f'(x)=5x^4-\sqrt{x}$ $=5x^4-x^{\frac{1}{2}}$ | $f(x)=x^5-\frac{2}{3}x^{\frac{3}{2}}+C$ |
| 6. $f'(x)=2\sin x$ | $f(x)=-2\cos x+C$ |
| 7. $f'(x)=3\cos x-e^x$ | $f(x)=3\sin x-e^x+C$ |

Summarize the strategies to do above problems:

Why $+C$ ?

$\Rightarrow \frac{dC}{dx}=0 \Rightarrow \therefore$ To find $f(x)$ from $f'(x)$, you need to add an arbitrary $C$ constant.

- **Antidifferentiation**: The process of finding $f(x)$ from $f'(x)$ ($\frac{df}{dx}$) is called "antidifferentiation", "antiderivative" or "integration",

$f(x) \Leftarrow$ correction needs to be made.

- **Notation of Antidifferentiation (Integration);**

$$\textbf{Given } \frac{df}{dx}, \textbf{ find } f(x) \quad \Leftrightarrow \quad f(x)=\int\left(\frac{df}{dx}\right)dx$$

- **Antidifferentiation rule similar to the power rule of differentiation.**

$$\int(ax^n)dx=a\frac{x^{n+1}}{n+1}+c, \textbf{ where } n\neq 1$$

**Examples) Integrate the followings:**

**1.** $\int\left(2x^2+\frac{1}{2}x\right)dx=\boxed{\frac{2}{3}x^3+\frac{1}{4}x^2+C}$

**2.** $\int\left(2x^5+\sqrt{x}\right)dx=\int\left(2x^5+x^{\frac{1}{2}}\right)dx$

$=\frac{2}{6}x^6+\frac{2}{3}x^{\frac{3}{2}}+C$

$=\boxed{\frac{1}{3}x^6+\frac{2}{3}x\sqrt{x}+C}$

**3.** $\int(\cos 5x)dx=\boxed{\frac{1}{5}\sin 5x+C}$

**2.** $\int\left(\frac{1}{x^2}+\frac{1}{x}\right)dx=$

$=\int\left(x^{-2}+x^{-1}\right)dx=\boxed{\frac{-1}{x}+\ln x+C}$