IB Math 3: Sequences and Series Name: Key Period: _____

Warn Up:

Evaluate $\int_4^{\infty} \frac{x\,dx}{\sqrt[3]{x-2}} = \lim_{a\to\infty} \int_4^{a} x(x-2)^{-\frac{1}{3}}dx$

$u = x-2 \quad x = u+2$

$du = dx$

$x \to \infty \quad u \to \infty$

$x = 4 \quad u = 2$

$\lim_{a\to\infty} \int_2^{a} (u+2)u^{-\frac{1}{3}}du$

$= \lim_{a\to\infty} \int_2^{a} \left(u^{\frac{2}{3}} + 2u^{-\frac{1}{3}}\right)du = \lim_{a\to\infty} \left[\frac{3}{5}u^{\frac{5}{3}} + (2)\left(\frac{3}{2}\right)u^{\frac{2}{3}}\right] = \infty$

diverges

Infinite Series: $S_n = a_1 + a_2 + a_3 ------- = \sum_{n=0}^{\infty}(a_n)$

1. Geometric Series:

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{a_1(1-r^n)}{1-r} = \frac{a_1}{1-r}$ where $|r| < 1$

$\lim_{n\to\infty} S_n = \textit{diverges}$ where $|r| \geq 1$

Example 1) Show that the series $\sum_{n=1}^{\infty}\left(\frac{1}{5^n}\right)$ converges.

$\sum_{n=1}^{\infty}\left(\frac{1}{5^n}\right) = \frac{1}{5} + \frac{1}{25} + \frac{1}{125} \cdots \Rightarrow r = \frac{1}{5} \Rightarrow S = \frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \boxed{\frac{1}{4}}$

Example 2) For the series $\sum_{n=0}^{\infty}\left(\frac{2^{n+1}}{5^n}\right)$, Find the Sum if it converges.

$\sum_{n=0}^{\infty}\left(2\cdot\left(\frac{2}{5}\right)^n\right) \Rightarrow$ G Series $r = \frac{2}{5}$ $a_1 = 2$

$S = \frac{2}{1-\frac{2}{5}} = \frac{2}{\frac{3}{5}} = \boxed{\frac{10}{3}}$

2. Sigma Notation:
Write each series in sigma notation:

$\Rightarrow$ a. $20 - (-5) + \cancel{\frac{4}{5}}\,\frac{5}{4} - (\frac{-5}{16}) \cdots\cdots\cdots + (\frac{-5}{256})$

b. $\frac{1}{1\cdot 2} + \frac{5}{2\cdot 3} + \frac{9}{3\cdot 4} \cdots\cdots\cdots + \frac{37}{10\cdot 11}$

There are typos!

$a_n = (20)\left(-\frac{1}{4}\right)^{n-1}$ OR $(-80)\left(-\frac{1}{4}\right)^n$

$\sum_{n=1}^{6}(20)\left(-\frac{1}{4}\right)^{n-1}$

$a_n = \frac{4n-3}{n(n+1)}$

$\sum_{n=1}^{10} \frac{4n-3}{n(n+1)}$

2. Telescoping Series: $\lim_{n\to\infty} S_n = (b_1 - b_2) + (b_2 - b_3) + (b_3 - b_4) - - - - - - - = b_1 - \lim_{n\to\infty} b_{n+1}$

Notice b2 is canceled by the second term, b2, b3 is canceled by the third term and so on. So a telescoping series will converge if $\lim_{n\to\infty} b_{n+1}$ **converges.**

Example 1) Given the series $\sum_{k=1}^{\infty}\left(\frac{2}{4k^2-1}\right)$;

a. Use partial fractions express; $\frac{2}{4k^2-1} = \frac{A}{2k+1} + \frac{B}{2k-1}$.   A=1   B = -1

b. Write $S_n = \sum_{k=1}^{n} \frac{A}{2k-1} + \frac{B}{2k+1}$ into the telescoping form.

$$= \left[\left(\frac{1}{1} + \cancel{\frac{-1}{3}}\right) + \left(\cancel{\frac{1}{3}} + \cancel{\frac{-1}{5}}\right) + \left(\cancel{+\frac{1}{5}} + \cancel{\frac{-1}{7}}\right) \cdots \left(\cancel{\frac{1}{2n-1}} - \frac{1}{2n+1}\right)\right]$$

c. Express $\lim_{k\to\infty} S_n = \sum_{i=1}^{\infty} \frac{A}{2k-1} + \frac{B}{2k+1} = \lim_{k\to\infty}(\quad)$ and evaluate the limit to find the sum.

$$\Rightarrow \lim_{k\to\infty} S_n = \lim_{k\to\infty}\left[1 - \frac{1}{2k+1}\right] = \boxed{1}$$

Practice) Evaluate the telescoping series, $\sum_{k=1}^{\infty}\left(\frac{1}{k^2+k}\right)$,for the convergence and find the sum if it converges.

$$\sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \sum_{k=1}^{\infty}\left[\frac{A}{k} + \frac{B}{k+1}\right] \qquad A=1 \quad B=-1$$

$$= \lim_{k\to\infty}\left[\left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) \cdots \cancel{\frac{1}{k}} - \frac{1}{k+1}\right]$$

$$= \lim_{k\to\infty}\left[1 - \frac{1}{k+1}\right] = \boxed{1}$$