IB Math HL2: Geometric Power Series Name: ______________ Period: _______

Warm up

Given $f(x)=\sum_{n=1}^{\infty}(\frac{x^n}{n})=x+\frac{x^2}{2}+\frac{x^3}{3}$..........

a) Find $\int f(x)dx$ **and write the resulting series in sigma notation.**

$$\int f(x)dx = \frac{x^2}{2\cdot 1}+\frac{x^3}{3\cdot 2}+\frac{x^4}{4\cdot 3}+\frac{x^5}{5\cdot 4}\cdots\cdots = \sum_{n=1}^{\infty}\frac{x^{n+1}}{(n+1)\cdot n}$$

b) Determine the radius of convergence for $\int f(x)dx$.

Ratio test:

$$\lim_{n\to\infty}\left[\frac{x^{n+2}}{(n+2)\cancel{(n+1)}}\right]\cdot\left[\frac{n\cancel{(n+1)}}{x^{n+1}}\right] = \lim_{n\to\infty}\left(\frac{\cancel{x^{n+1}}\cdot x\cdot}{\cancel{x^{n+1}}}\right)\left(\frac{n}{n+2}\right)=x.$$

$|x|<1$, $R=1$.

Geometric power Series

$$\sum_{n=1}^{\infty} ar^n=\frac{a}{1-r} \text{ where } |r|<1$$

Example 1) Find a power series for $f(x)=\frac{4}{x+2}$ **centered at x=0. And find the radius of convergence.**

Solution:

$$f(x)=\frac{4}{x+2}=\frac{(4)}{2+x}=\frac{4 \div 2}{2-(-x)\div 2}=\frac{2}{1-(-\frac{x}{2})}$$

$\Rightarrow |r|=|-\frac{x}{2}|$

$\Rightarrow a=2$

$$f(x)=(2)+(2)(-\tfrac{x}{2})+(2)(-\tfrac{x}{2})^2+(2)(-\tfrac{x}{2})^3\cdots$$

$$\frac{4}{x+2}=\sum_{n=0}^{\infty}(2)(-1)^n\left(\frac{x}{2}\right)^n$$

Interval of convergence

$|\frac{-x}{2}|<1 \Rightarrow \boxed{-2<x<2}$

$$\frac{a}{1-r}.$$

Example 1) Find a power series for $f(x)=\frac{1}{x}$ centered at x=1. And find the radius of convergence.

Solution:

$$f(x)=\frac{1}{x}=\frac{1}{1+(x-1)}=\frac{1}{1-[-(x-1)]}$$

$$r=|-(x-1)|=|x-1|$$

$$=\sum_{n=0}^{\infty}[-(x-1)]^n$$

$$|x-1|<1 \Rightarrow R=1.$$

$$=\sum_{n=0}^{\infty}(-1)^n(x-1)^n$$

$$0<x<2.$$

$$\frac{a}{1-r}$$

Practice) Find a power series for $f(x)=\frac{3}{5+2x}$ centered at x=1. And find the radius of convergence.

$$f(x)=\frac{3}{5+2(x-1)+2}=\frac{3}{7-[-2(x-1)]}=\frac{\frac{3}{7}}{1-[-\frac{2}{7}(x-1)]}$$

$$=\sum_{n=0}^{\infty}\frac{3}{7}(-1)^n\left[\frac{2}{7}(x-1)\right]^n$$

$$\left|-\frac{2}{7}(x-1)\right|<1$$

$$-\frac{7}{2}<x-1<\frac{7}{2}$$

Interval: $\boxed{-\frac{5}{2}<x<\frac{9}{2}}$

$$R=\frac{7}{2}.$$

Notes: Given $f(x)=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}$..........

a) Show $f'(x)=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}$..........

b) Show $\int f(x)dx=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}$..........

c) What is the function which has the property of $f(x)=f'(x)=\int f(x)dx$?