IB Math HL2 Homogeneous Equations:

$\frac{dy}{dx} = F(\frac{y}{x})$ $\quad y = vx$ (substitution) $\quad v = \frac{y}{x}$.

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\Rightarrow$ $\boxed{v + x\frac{dv}{dx} = F(v)}$ (Solve by separating variables).

$\rightarrow x \cdot \frac{dv}{dx} = F(v) - v$

$\Rightarrow \frac{x}{dx} = \frac{F(v) - v}{dv}$

$\Rightarrow \frac{dx}{x} = \frac{dv}{F(v) - v}$.

Example 1) $(x^2 - y^2)dx + 3xydy = 0$ (divided by x^2)

$\Rightarrow (1 - \frac{y^2}{x^2})dx + (\frac{3y}{x})dy = 0$ (divided by dx)

$\Rightarrow (1 - \frac{y^2}{x^2}) + 3(\frac{y}{x})\frac{dy}{dx} = 0$

$\Rightarrow (1 - v^2) + 3 \cdot v(v + x \cdot \frac{dv}{dx}) = 0$

$\Rightarrow 1 - v^2 + 3v^2 + 3v \cdot dv \cdot (\frac{x}{dx}) = 0$

$\Rightarrow (1 + 2v^2) + 3v\,dv\,(\frac{x}{dx})$

$3v \cdot dv(\frac{x}{dx}) = -(1 + 2v^2)$

$\frac{x}{dx} = \frac{-(1+2v^2)}{3v\,dv}$.

$\int \frac{dx}{x} = \int \frac{-3v\,dv}{1+2v^2}$

$u = 1 + 2v^2$, $\frac{1}{4}du = v\,dv$ (?)

$\ln x = -\frac{3}{4}\ln(1 + 2v^2) + C$.

Wildlife Population)

The rate of change of the number of coyotes, N(t), in population is directly proportional to 650-N(t), where t is the time in years. When t=0, the population is 300 and t=2 the population has increased to 500.

a. Set up the differential equation to model the population of coyotes, N(t).
b. Find the population when t=5.

a) $\frac{dN}{dt} = k(650 - N)$ (N: population, t: yrs).

b) $\int \frac{dN}{650 - N} = \int k\,dt$

$-\ln(650 - N) = kt + C$

$\ln(650 - N) = -kt - C$

$650 - N = e^{-kt-C} = A \cdot e^{-kt}$

$A = e^{-C}$

$N = 650 - A \cdot e^{-kt}$

$t = 0 \quad N = 300$

$300 = 650 - A \cdot e^0$

$A = 350$

$N = 650 - 350e^{-kt}$

$N = 650 - 350e^{(\ln\frac{3}{7}/2)t}$

$t = 2 \quad N = 500$

$500 = 650 - 350e^{-2k}$

$e^{-2k} = \frac{150}{350} = \frac{3}{7}$.

$k = (\ln\frac{3}{7})/-2$

$= \ln\frac{3}{7}/2$