IB Math HL2 Homogeneous Equations:

$\frac{dy}{dx} = F(\frac{y}{x})$ $\quad y = vx$ (substitution) $\quad v = \frac{y}{x}.$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\Rightarrow$ $\boxed{v + x\frac{dv}{dx} = F(v)}$ (Solve by separating variables).

Example one) $\underline{(x^2+y^2)dx + 2xydy = 0}$
$dx \qquad dx$

$v = \frac{y}{x}. \Rightarrow y = xv$

$\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}$

$$\frac{(x^2+y^2) + 2xy\cdot\frac{dy}{dx}}{x^2} = \frac{0}{x^2}$$

$$\Rightarrow \left(1 + \left(\frac{y}{x}\right)^2\right) + \frac{2y}{x}\cdot\frac{dy}{dx} = 0$$

Continue on attached.

Example two) $\underline{(x+y)dy + (x-y)dx = 0}$
$dx \qquad dx$

$v = \frac{y}{x}. \quad y = x\cdot v$

$\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}.$

$$\frac{(x+y)\cdot\frac{dy}{dx} + (x-y)}{x} = \frac{0}{x}.$$

$$\left(1 + \frac{y}{x}\right)\cdot\frac{dy}{dx} + \left(1 - \frac{y}{x}\right) = 0.$$

$$(1+v)\left[v + x\cdot\frac{dv}{dx}\right] + \underline{(1-v)} = 0$$

$$\frac{\cancel{(1+v)}\left(v + x\cdot\frac{dv}{dx}\right)}{\cancel{1+v}} = \frac{v-1}{v+1}$$

②

$$\Rightarrow (1+v^2) + 2v\left(v + x\cdot\frac{dv}{dx}\right) = 0$$

$$\frac{\cancel{2}v\left(v + x\frac{dv}{dx}\right)}{2\cancel{v}} = \frac{-(v^2+1)}{2v}$$

$$\cancel{v} + \frac{x\cdot dv}{dx} = \frac{-(v^2+1)}{2v} - v$$

$$\frac{x}{dx}\cdot dv = \frac{-v^2-1-2v^2}{2v}$$

$$\frac{x}{dx} = \frac{-3v^2-1}{(2v)\,dv}$$

$$\int\frac{dx}{x} = \int\frac{-(2v)\cdot dv}{3v^2+1.}$$

$u = 3v^2+1$

$\frac{1}{6}du = v\,dv$

$$\ln x = \frac{-2}{6}\ln(3v^2+1) + C$$

$$\boxed{\ln x + \frac{1}{3}\ln\left(3\left(\frac{y}{x}\right)^2+1\right) = C}$$

$$\ln\left[x\cdot\sqrt[3]{3\left(\frac{y}{x}\right)^2+1}\right) = C.$$

$$v + \frac{x \cdot dv}{dx} = \frac{v-1}{v+1} - v$$

$$-v$$

$$\frac{x}{dx} \cdot dv = \frac{v-1-v(v+1)}{v+1}$$

$$\frac{x}{dx} = \frac{\cancel{v}-1-v^2-\cancel{v}}{(v+1)\cdot dv}.$$

$$\int \frac{dx}{x} = \int \frac{-(v+1)\,dv}{v^2+1}$$

$$\Rightarrow \ln x = \int \frac{-v}{v^2+1}\,dv - \int \frac{1}{v^2+1}\,dv.$$

$$\Rightarrow \ln x = \frac{-1}{2}\ln(v^2+1) - \arctan v + C.$$

$$\boxed{\ln x + \ln\sqrt{\left(\frac{y}{x}\right)^2+1} + \arctan\left(\frac{y}{x}\right) = C}$$