key



2015 IB paper 3 Question.

$\frac{dy}{dx} = \frac{y^2+x^2}{2x^2}$ for which $y=-1$ when $x=1$.

$dx = h = 0.25 \Rightarrow x_{n+1} = x_n + 0.25 \Rightarrow y(2) = ?$

$y_{n+1} = y_n + y' \cdot dx$.

$[y_{n+1} = y_n + F(x_n, y_n) \cdot h]$

| $n$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $x_n$ | 1 | 1.25 | 1.5 | 1.75 | 2.0 |
| $y_n$ | -1 | -0.750 | -.580 | -.436 | -0.303 |

$\Rightarrow y(2) \approx -.303$.

$$y_1 = -1 + \left[\frac{(-1)^2 + (1)^2}{2(1)^2}\right] \cdot 0.25 = -0.75$$

$$y_2 = -0.75 + \left[\frac{(-0.75)^2 + (1.25)^2}{2(1.25)^2}\right] \cdot 0.25 = -.58\ldots$$

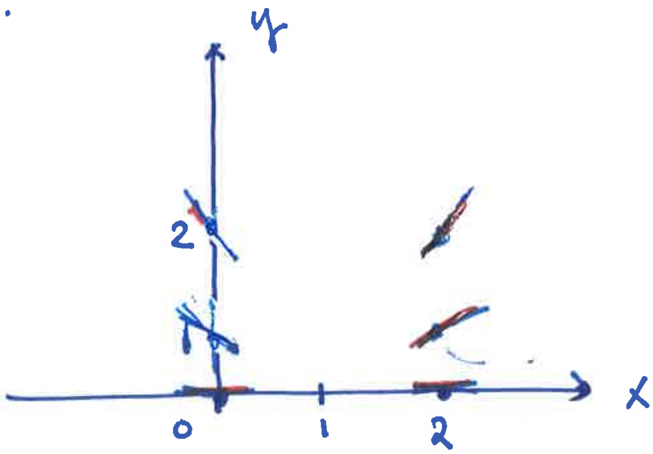
$$y_3 = -.58 + \left[\frac{(-0.58)^2 + (1.5)^2}{2(1.5)^2}\right] \cdot 0.25 \approx -.4363 \approx -.436$$

$$y_4 = -.436 + \left[\frac{(-.436)^2 + (1.75)^2}{2(1.75)^2}\right] \cdot 0.25 \approx -.303$$

Ap #4.

②

a)




Sample calculations:

$$\frac{dy}{dx} = \frac{y^2}{x-1}$$

at (2,1)

$$\frac{dy}{dx} = \frac{(1)^2}{(2)-1} = \boxed{\frac{1}{1}} = \boxed{1}.$$

at (2,0)

$$\frac{dy}{dx} = \frac{0}{(2)-1} = \boxed{0}$$

at (0,1)

$$\frac{dy}{dx} = \frac{1}{0-1} = -1$$

b) $f(2) = 3$

$$\frac{dy}{dx} = \frac{y^2}{x-1} = \frac{3^2}{2-1} = 9.$$

Tangent line: $\boxed{y-3 = 9(x-2)} \Rightarrow \boxed{y = 9x-15}$

$$y(2.1) = 9(2.1)-15 \approx \boxed{3.9}$$

c) $\frac{dy}{dx} = \frac{y^2}{x-1}. \Rightarrow \int \frac{dy}{y^2} = \int \frac{dx}{x-1} \Rightarrow -\frac{1}{y} = \ln|x-1| + C.$

$$y = \frac{-1}{\ln|x-1|+c} \Leftarrow (2,3)$$

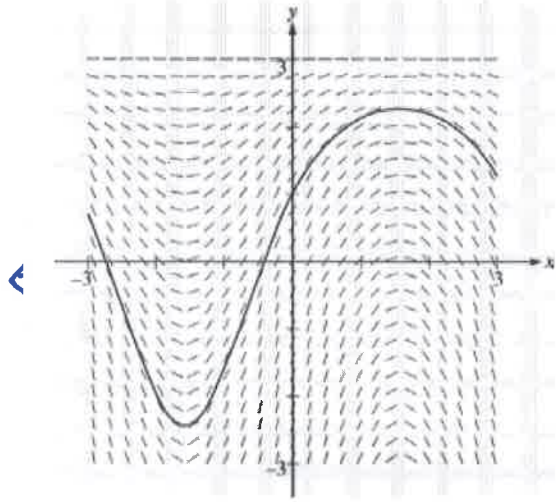
$$3 = \frac{-1}{\ln(1) + c} \Rightarrow 3 = \frac{-1}{c} \Rightarrow c = \frac{-1}{3}$$

$$y = \frac{-1 \quad (-3)}{\ln|x-1| - \frac{1}{3} \ (-3)} = \boxed{\frac{+3}{1 - 3\ln|x-1|}}$$

$$f(x) = \frac{3}{1 - 3\ln|x-1|}$$

Ap #6.

a)



b) $\frac{dy}{dx} = (3-y)\cos x$

$(0,1) \Rightarrow \frac{dy}{dx} = (3-1)\cos 0 = 2$

$\Rightarrow y = 2x+1 \Rightarrow f(0.2) = 2(0.2)+1 = \boxed{1.4}$

c) $\int \frac{dy}{3-y} = \int \cos x \, dx$

$-\ln|3-y| = \sin x + C. \Leftarrow (0,1)$

$C = -\ln 2.$

$\Rightarrow -\ln|3-y| = \sin x - \ln 2$

$\ln|3-y| = \ln 2 - \sin x$

$3-y = e^{\ln 2 - \sin x}.$

$y = 3 - e^{\ln 2 - \sin x}.$

$y = 3 - 2 \cdot e^{-\sin x}$