IB Exam Questions. Answers.

#1. a) $E(X) = \int_0^{\frac{\pi}{2}} x \sin x \, dx$

u	dv
x	$\sin x$
1	$-\cos x$
0	$-\sin x$

$= \left[-x\cos x + \sin x\right]_{x=0}^{x=\frac{\pi}{2}}$

$= \boxed{1}$

#2. Normal Distribution

$\mu = 11 \quad \sigma = 3$

(a) $P(X > 15) \Rightarrow$ Norcdf $(15, 999, 11, 3) = 0.0912$.

(b) Binomial : $X \sim B(5, 0.0912)$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - \binom{5}{0}(0.0912)^0(1-.0912)^5 = \boxed{0.380}$

#3. $\lambda = \frac{1}{4}$ $\quad X$: #s of cars that arrive in T min.

$X \sim Po(0.25T)$

(a) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= e^{-\mu}\left[\frac{\mu^0}{0!} + \frac{\mu^1}{1} + \frac{\mu^2}{2} + \frac{\mu^3}{6}\right] = 0.6$

use Graphing cal. Solve for μ $\quad \mu = 0.25T \Rightarrow T = 13$ min.

(b) $T = 10$ min. $\quad \mu = \frac{10}{4} = \frac{5}{2} = 2.5$ minutes.

$\Rightarrow$ 6 cars can cross. $\quad X_1$: within 10 min. $\quad X_2$: Within Next 10 min

$P(\text{All get on}) = P(X_1 \leq 3)P(X_2 \leq 3) + P(X_1 = 4)P(X_2 \leq 2)$

$\Rightarrow [\text{pocdf}(2.5,3)] + \text{popdF}(2.5,4)\,\text{pocdF}(2.5,2)$ (3

$+ \text{popdF}(2.5,5)\,\text{pocdF}(2.5,1) + \text{popdF}(\cancel{2}5,6)\,\text{popdF}(2.5,$

$= \boxed{0.668}$

#4. (a) InvNor ~~Norcdf~~ $(\cancel{z} > k_1) = 0.25$

InvNor $(z < k_1) = 0.75$

InvNor $(z < k_2) = 0.25$

$\Rightarrow$ $k_1 = \dfrac{21.3-\mu}{\sigma} = 0.674$

$k_2 = \dfrac{17.1-\mu}{\sigma} = -0.67$

$\mu = \dfrac{21.3+17.1}{2} = 19.2\,\text{kg}.$ $\sigma = 3.11\,\text{kg}.$

$E(x) = 100 \cdot p.$ $(X \sim B(100, p))$ $p \approx 0.18424.$

b) $100\,p(X > 22) = 100$ Normal cdF $(22, 999, 19.2, 3.11)$

$= (100)(0.18424) = \boxed{18.4}$

#5 (a) $E(x) = \int_0^1 12x^3(1-x)\,dx = 12\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1 = \frac{3}{5}$

(b) $F'(x) = 12(2x-3x^2) = 0$ $\boxed{x = \frac{2}{3}}$

#6. (a) $2.1 = \mu - 0.524\sigma$ $p(X > 2.1) = .70$

$2.5 = \mu + 0.674\sigma$ $P(X > 2.5) = .25$

$\mu = 2.27$ $\sigma = 0.334$

$X \sim B(10, .25)$

(b) (i) $P(X > 2.5) = .25$ $\Rightarrow$ $E(x) = (10)(.25) = 2.5.$

(b) (ii) $P(X=5) = \binom{10}{5}(.25)^5(.75)^5 = $ Binomial pdF (10, 0.25, 5)

$= 0.0584$

(iii) Mode (Most likely)

$P(X=0) = 0.056$

$P(X=1) = 0.1877$

$P(X=2) = 0.2815 \Leftarrow \boxed{X=2}$

$P(X=3) = 0.2502$

#6. (C) (i) $1 - P(y \le 1) = 0.80085$ $\quad \lambda = 3$. (mean)

$1 - P(y=0) - P(y=1) = 1 - \frac{e^{-\mu}\mu^0}{0!} - \frac{e^{-\mu}\mu^1}{1!} = 0.80083$

Solve by GC

(ii) X_1 : eggs by ~~1st~~ bird

X_2 : eggs by 2nd bird

$P(\text{Eggs}=2) = P(X_1=0)P(X_2=2) + P(X_1=1)P(X_2=1)$

$+ P(X_1=2)P(X_2=0)$

$= (e^{-3})\frac{(3)^2}{2!}(e^{-3}) + \left(e^{-3}\frac{(3)^1}{1!}\right)^2 + (e^{-3})\left(\frac{3^2}{2}\right)(e^{-3})$

$= \boxed{0.0446}$

(iii) $P(X_1=1 \text{ and } X_2=1 \mid X_2+X_1=2)$

$= \frac{e^{-3}(3)^2}{\ldots} = \boxed{0.5}$

#7. $X \sim Po(\mu)$

$$P(X=10) = 2P(X=9)$$

$$= \frac{e^{-\mu}\cdot\mu^{10}}{10!} = \frac{2e^{-\mu}\mu^{9}}{9!} \Rightarrow \boxed{\mu = 20}$$

$$\boxed{E(X) = 20}$$

#8. (a) $k\displaystyle\int_0^1 \frac{dx}{\sqrt{4-x^2}} = 1 \Rightarrow k \arcsin \frac{x}{2}\Big]_0^1 = 1$

$$k\left[\arcsin\left(\tfrac{1}{2}\right) - \arcsin 0\right] = 1$$

$$k \cdot \frac{\pi}{6} = 1 \quad \boxed{k = \frac{6}{\pi}}$$

(b) $E(X) = \dfrac{6}{\pi}\displaystyle\int_0^1 \frac{x\,dx}{\sqrt{4-x^2}}$

$$u = 4 - x^2 \qquad \frac{du}{dx} = -2x \Rightarrow du = -2x\,dx.$$

$$\left.\begin{matrix} x=0 & u=4 \\ x=1 & u=3 \end{matrix}\right) \Rightarrow \frac{6}{\pi}\int_4^3 \left(\frac{-1}{2}\right) u^{-\frac{1}{2}}\,du.$$

$$= \boxed{\frac{6}{\pi}\left(2 - \sqrt{3}\right) = E(X)}$$

(c)

$$\frac{6}{\pi}\int_0^m \frac{dx}{\sqrt{4-x^2}} = \frac{1}{2}.$$

$$= \frac{6}{\pi}\left[\arcsin\left(\frac{x}{2}\right)\right]_0^m = \frac{1}{2} \qquad \arcsin\left(\frac{m}{2}\right) = \frac{\pi}{12}$$

[illegible]

#9

(a) $P(3 \le X \le 5) = 0.547$

$\text{PoCdF}(4, 3, 5) = 0.547$

(b) $P(X \ge 3) = 1 - P(X \le 2) = 0.762$

(c) $P(X \le X \le 5 / X \ge 3) = \dfrac{0.547}{0.762} = 0.718$

#10. $P(X > 90) = 0.15$ and $P(X < 40) = 0.12$

$1.636 = \dfrac{90 - \mu}{\sigma}$ and $-1.175 = \dfrac{40 - \mu}{\sigma}$

$\mu = 66.6$ and $\sigma = 22.6$

#11 Weight of glass = X

$X \sim N(160, \sigma^2)$

$P(X < 174) = 0.75 \Rightarrow \dfrac{X - 160}{\sigma} = 0.75$ $\sigma = 20.8$

#12. Key attached

IB Question #12

Key

12a) i) $X \sim Po(11)$

$P(X \leq 11) \rightarrow$ poissoncdf$(2.2 \times 5, 11)$

poissoncdf$(11, 11)$

$= \boxed{0.579}$

ii) $P(X > 8 \mid x < 12)$

$= \dfrac{P(X=9) + P(X=10) + P(X=11)}{P(X<12)}$

$= \dfrac{\text{poissonpdf}(11,9) + \text{poissonpdf}(11,10) + \text{poissonpdf}(11,11)}{\text{poisson}(11,11)}$

$= 0.59979$

$= \boxed{0.6}$

b) i) $P(Y > 3) = 0.24$

$\therefore P(Y \leq 3) = 0.76$

$\rightarrow P(X = x) = \dfrac{e^{-\mu} \mu^x}{x!}$

$P(X=0) + P(X=1) + P(X=2) + P(X=3) = 0.76$

$\therefore e^{-\mu}\left[\dfrac{\mu^0}{0!} + \dfrac{\mu^1}{1!} + \dfrac{\mu^2}{2!} + \dfrac{\mu^3}{3!}\right] = 0.76$

$e^{-\mu}\left[1 + \mu + \dfrac{1}{2}\mu^2 + \dfrac{1}{6}\mu^3\right] = 0.76$

$\boxed{\mu = 2.49}$

graph on graphing calculator $x = 2.49$ when $y = 0.76$

ii) $A \sim Po(4.98)$ 2 wknds $\rightarrow 2(2.49) = 4.98$

$P(A > 5) = 1 - P(A \leq 5)$

$= 1 -$ poissoncdf$(4.98, 5)$

$= 0.38053$

$\rightarrow W \sim B(4, 0.38)$

4 wknds in Feb

$\rightarrow P(W \geq 2)$ (more than 5 accidents during at least 2 wknds)

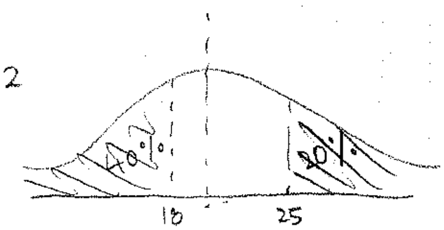
$= 1 - P(W \leq 1)$

$= \boxed{0.490}$

c) $P(A < 25) = 1 - 0.2$ since $P(A \geq 25) = 0.2$

$= 0.8$

$P(A < 18) = 0.4$



$\rightarrow \dfrac{25 - \mu}{\sigma} = 0.8416$

$\dfrac{18 - \mu}{\sigma} = -0.2533$

$\dfrac{25 - \mu}{0.8416} = \dfrac{18 - \mu}{-0.2533}$

$\boxed{\mu = 19.6}$

$\rightarrow$ 2nd vars $\rightarrow$ invnorm (0.6)