ıme: ______________ Group Members: ______________ Date: ______

xploration 7-2a: Differential Equation ɔr Compound Interest

bjective: Write and solve a differential equation for the amount of money in a savings count as a function of time.

ıen money is left in a savings account, it earns interest ual to a certain percent of what is there. The more ɔney you have there, the faster it grows. If the interest *compounded continuously,* the interest is added to the count the instant it is earned.

1. For continuously compounded interest, the instantaneous rate of change of money is directly proportional to the amount of money. Define variables for time and money, and write a differential equation expressing this fact.

2. Separate the variables in the differential equation in Problem 1, then integrate both sides with respect to t. Transform the integrated equation so that the amount of money is expressed explicitly in terms of time.

3. The integrated equation from Problem 2 will contain e raised to a power containing *two* terms. Write this power as a product of two different powers of e, one that contains the time variable and one that contains no variable.

4. You should have the expression e^C in your answer to Problem 3. Explain why e^C is always positive.

5. Replace e^C with a new constant, C_1. If C_1 is allowed to be positive or negative, explain why you no longer need the ± sign that appeared when you removed the absolute value in Problem 2.

6. Suppose that the amount of money is \$1000 when time equals zero. Use this initial condition to evaluate C_1.

7. If the interest rate is 5% per year, then $d(\text{money})/d(\text{time}) = 0.05(\text{money})$ in dollars per year. What, then, does the proportionality constant in Problem 1 equal?

8. How much money will be in the account after 1 year? 5 years? 10 years? 50 years? 100 years? Do the computations in the most time-efficient manner.

9. How long would it take for the amount of money to double its initial value?

lame: ______________ Group Members: ______________ Date: ______

Exploration 7-3a: Differential Equation or Memory Retention

Objective: Write and solve a differential equation for the number of names remembered s a function of time.

a Member is a freshman at a large university. One vening he attends a reception at which there are many nembers of his class whom he has not met. He wants to redict how many new names he will remember at the end f the reception.

1. Ira assumes that he meets people at a constant rate of R people per hour. Unfortunately, he forgets names at a rate proportional to y, the number he remembers. The more he remembers, the faster he forgets! Let t be the number of hours he has been at the reception. What does dy/dt equal? (Use the letter k for the proportionality constant.)

2. The equation in Problem 1 is a differential equation because it has differentials in it. By algebra, separate the variables so that all terms containing y appear on one side of the equation and all terms containing t appear on the other side.

3. Integrate both sides of the equation in Problem 2. You should be able to make the integral of the reciprocal function appear on the side containing y.

4. Show that the solution in Problem 3 can be transformed into the form

   $ky = R - Ce^{-kt}$

   where C is a constant related to the constant of integration. Explain what happens to the absolute value sign that you got from integrating the reciprocal function.

5. Use the initial condition $y = 0$ when $t = 0$ to evaluat the constant C.

6. Suppose that Ira meets 100 people per hour, and that he forgets at a rate of 4 names per hour when $y = 10$ names. Write the particular equation expressing y in terms of t.

7. How many names will Ira have remembered at the end of the reception, $t = 3$ h?

IB Math 2: More Practice of Differential Equations: Name: _______________ Period: ______

1. Find the solution to the differential equation.

a. $\frac{dy}{dx} = 4y^2$, and x=1, y=2

b. $\frac{dy}{dx} = 8x^2y^2$, and x=1. y=2/3

c. $\frac{dy}{dx} = \frac{\sin x}{ye^y}$, and $y(0) = 0$. Solve for y in terms of x.

2. The rate of change of y is proportional to y. When t=0, y=2, and when t=2, y=4. What is the value of y when t=3?

3. The rate of growth of the volume of a sphere is proportional to its volume. If the volume of the sphere is initially $36\pi \quad ft^3$, and expands to $90\pi \quad ft^3$ after 1 second, find the volume of the sphere after 3 seconds.

4. The rate of decay of a radioactive substance is proportional to the amount of substance, S, present at time t years. The proportional constant is -0.1152.

a. Set up a differential equation involving S and t.
b. Find the half-life of the substance.
c. How much substance remains after 20 years.